Zbl 013.24603
Erdős, Paul
On the integers which are the totient of a product of three primes. (In English)
Q. J. Math., Oxf. Ser. 7, 16-19 (1936).

The main result of this paper is that if $f(n)$ is the number of representations of n as $(p-1)(q-1)(r-1)$ where p, q, r are different primes, then $\overline{\lim}_{n\to\infty} f(n) = \infty$. The proof is based on a slightly more precise form of the proposition (established by the author in a previous paper, Zbl 012.14905) that for almost all primes $p \leq n$, $p-1$ has between $(1-\varepsilon)\log\log n$ and $(1+\varepsilon)\log\log n$ different prime factors. He is then able to prove that, if $p'q'r'$ denote primes with $p'q'r' \leq n$ for which $p'-1$, $q'-1$, $r'-1$ have each more than $(1-\varepsilon)\log\log n$ different prime factors, then the number of different number of the form $(p'-1)(q'-1)(r'-1)$ is of lower order of magnitude than the number of sets $p'q'r'$. This establishes the main result. Correction. The formula between (2) and (3) should read:

$$N(P_k, n) < \frac{C_n}{\log^2 n}\frac{(C+\log\log n)^{k+3}}{(k-1)!} + o\left(\frac{n}{\log^2 n}\right)$$

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Davenport

Classification:
11A25 Arithmetic functions, etc.
11A41 Elemementary prime number theory