Zbl 012.14905
Erdős, Paul
On the normal number of prime factors of $p-1$ and some related problems concerning Euler's φ-function. (In English)
Q. J. Math., Oxf. Ser. 6, 205-213 (1935).

The main results of this paper are as follows.
I. The normal number of (different) prime factors of $p-1$ (where p is a prime) is $\log\log n$, i.e. if $\varepsilon > 0$ is given, then for all but $o(n/\log n)$ primes $p \leq n$, the number of prime factors of $p-1$ lies between $(1-\varepsilon)\log\log n$ and $(1+\varepsilon)\log\log n$.
II. The number of integers $m \leq n$ which are representable as $\varphi(m')$ (where φ is Euler's function) is $O(n(\log n)^{\varepsilon-1})$, for any $\varepsilon > 0$
III. There exist infinitely many integers m which are representable as $\varphi(m')$ in more than m^C ways, where C is an absolute constant.
For the proof of I, an upper bound for the number of primes $p \leq n$ for which $(p-1)/a$ is a prime is obtained by Brun's method, and from it is deduced an upper bound for the number of primes $p \leq n$ for which $p-1$ has exactly k prime factors. — For II, the author succeeds in dividing the integers m' with $\varphi(m') \leq n$ into two classes, in such a way that the first class contains only $O(n/(\log n)^{\varepsilon-1})$ numbers m', and that for the second class $\varphi(m')$ has at least $20\log\log n$ prime factors and so can be shown to assume only $o(n/\log n)$ different values. — The proof of III cannot be summarised here.

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Classification:
11N25 Distribution of integers with specified multiplicative constraints
11A25 Arithmetic functions, etc.