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Erdős, Pál; Szekeres, George
Über die Anzahl der Abelschen Gruppen gegebener Ordnung und über ein verwandtes zahlentheoretisches Problem.
On the number of abelian groups of given order and on a related number-theoretical problem. (In German)
Acta Litt. Sci. Szeged 7, 95-102 (1934).

Let $f_i(n)$ denote the number of different (=disregarding the order of the factors) ways in which the integer n can be written as a product, each of whose factors is a prime number raised to a power $\geq i$. For the partial sums of these $f_i(n)$, the authors prove the asymptotic formulae $\sum_{k=1}^{n} f_i(k) = A_i n^{1/i} + O(n^{1/i+1})$; the constant $A_i = \prod_{k=1}^{\infty} \zeta(1 + k/i)$, where $\zeta(s)$ denotes Riemann's Zeta function. For $i = 1$, $\sum f_1(k)$ is the number of finite abelian groups whose orders are $\leq n$; it is $= A_1 \cdot n + O(n^{1/2})$. In a second part the authors give similar asymptotic formulae for the frequency of those integers for which $f_i(n) \neq 0$.

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11N45 Asymptotic results on counting functions for other structures
20K01 Finite abelian groups

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