VIOLATION OF THE STRONG EQUIVALENCE PRINCIPLE *

KOSTADIN TRENČEVSKI †
Institute of Mathematics,
Ss. Cyril and Methodius University, P. O. Box 1000,
Skopje, Macedonia
E-mail: kostatre@iunona.pmf.ukim.edu.mk

This paper at first considers the basic equivalence principles (Weak EP, Einstein EP and Strong EP), and further are presented some recent results about time dependent gravitational potential in the universe. Its consequences fit with the observations: Hubble red shift, change of the orbital period of the binary pulsars and anomaly acceleration of the spacecraft Pioneer 10 and 11. Section 3 considers the following experiment. A shielded laboratory is freely falling toward the Earth and assume that in the shielded laboratory there are two bodies which are moving under the mutual gravitation. Calculated is the quotient $\Theta_2 : \Theta_1$ according to the observer from the shielded laboratory of two successive orbital periods. Using the results about the time dependent gravitational potential, which are experimentally confirmed by the binary pulsars, $\Theta_2 : \Theta_1 \neq 1$. It violates the SEP, because according to SEP $\Theta_2 : \Theta_1 = 1$. The reason for this deviation from the General Relativity is explained. The last section considers a radial motion of a particle in a weak spherical gravitational field. Although in a short time interval the acceleration is almost a constant, it is shown that the GR equations are not close to the special relativity equations for motion under a constant force.

1. Equivalence principles and basic problems

The basic assumption in the General Relativity (GR) is the Weak Equivalence Principle (WEP), which states that if an uncharged test body is placed at an initial event in the space-time and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition. This principle requires too little, so every gravitational theory satisfies it. The first precise experiments of verification of

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WEP were done by Eötvös in 1922. Much more precise experiments were done later, and the best precision is achieved at Princeton (Roll, Krotkov and Dicke, 1964) and Moscow (Braginsky and Panov, 1972). These are laboratory tests, such that the relevant differences are in the test-body compositions. According to this principle the relevant test-body differences are their fractional nuclear-binding differences, their neutron-to-proton ratios, their atomic charges and so on. According to some recent experiments there is a slight deviation from the WEP. A University of Washington laboratory EP experiment [2] is designed to simulate the compositional differences of the Earth and Moon. That test of relative acceleration yields to $(1.0 \pm 1.4) \times 10^{-13}$, where systematic and random uncertainties are combined. Further, according to some recent results in Standard Model in the Particle Physics that contain new macroscopic-range quantum fields predict quantum exchange forces that will generically violate the WEP. Indeed, the particles couple to generalized "charges" rather than the mass/energy as in the gravity theory [3, 4].

Einstein in 1916 accepted in GR a stronger principle known as Einstein Equivalence Principle (EEP) [14]. This principle states that: (i) WEP is valid, (ii) the outcome of any local nongravitational test experiment is independent of the velocity of the (freely falling) apparatus, (iii) the outcome of any local nongravitational test experiment is independent of where and when in the universe it is performed. Local nongravitational test experiment means experiment (i) performed in any freely falling laboratory that is shielded and it is sufficiently small that inhomogeneities in the external fields can be ignored in the considered small volume, and (ii) in which self-gravitational effects are negligible. The EEP is the basic principle in the GR, because it implies that gravity must satisfy the postulates of Metric Theories of Gravity, i.e. (i) space-time is endowed with a metric g, (ii) the world lines of test bodies are geodesics of that metric, and (iii) in local freely falling frames, called local Lorentz frames, the nongravitational laws of physics are those of Special Relativity. Indeed, at each space-time point P we choose a freely falling coordinate system of arbitrary velocity, such that the metric at P can be reduced to Minkowskian, i.e. δ_{ij} (assuming imaginary time coordinate $x^4 = ict$). If we choose two such systems, then the Jacobi matrix will be a Lorentz transformation at P. At each point we can choose systems such as

$$g_{ij}(P) = \delta_{ij}, \quad \frac{\partial g_{ij}}{\partial x^k}(P) = 0.$$

Considering the space-time as a manifold endowed with metric, the well known techniques from the Riemannian geometry can be applied there.

The following questions appear. The previous consideration is correct from mathematical viewpoint, but from physical viewpoint only some special coordinates make sense. If we choose a coordinate system which is "convenient" for P, then it may not be "convenient" for the neighboring points. For example, in an inertial system with coordinates y^i we choose a rotating system with coordinates x^i according to the observer who rotates. Accepting any curvilinear system $x^i = x^i(y^1, y^2, y^3, y^4),\ i = 1, 2, 3, 4$ which corresponds to the observer who rotates, the geodesic lines which correspond according to x^1, x^2, x^3, x^4 are just straight lines according to the inertial system. But any such coordinate system is not physically convenient for the observer who rotates, because it is impossible for the Jacobi matrix $J = \frac{\partial(x^1,x^2,x^3,x^4)}{\partial(y^1,y^2,y^3,y^4)}$ to be a Lorentz transformation at each point as it is expected, because there is no gravitational field. Indeed, if J would be a Lorentz transformation everywhere, then the functional dependence must be a linear transformation and hence the Jacobi matrix must be a matrix with constant elements, which is a contradiction. A satisfactory solution of this problem was recently published [13].

We have a similar situation in case of the gravitational fields. The assumption that we deal with a 4-dimensional manifold, a priori means that there exists functional dependence between any two coordinate systems when they have joint interior points. This yields that the coordinate systems are not physically convenient, although the metric yields to the correct geodesic lines. Any such chosen coordinate system can only approximately give the physical reality, like the rotating system in an inertial frame. Hence it is natural to expect that the chosen mathematical apparatus in the GR gives satisfactory results according to the first post-Newtonian approximation (1PN). We shall see in the next sections how we can introduce new equations of motion which are "physically convenient".

The used mathematical apparatus is very convenient for calculation of the curvatures. So it is natural that the GR gives excellent result about the gravitational radiation. Also the metric proportions are convenient for the GR. This means that the GR explains the phenomena about gravitational red shift and the Shapiro time delay very well.

At the end of this section we shall consider the strong equivalence principle (SEP) [14]. In order to make stronger distinction among the metric theories of gravitation, introduced is the following SEP. It states that: (i) WEP is valid for self gravitating bodies as well as for the test bodies (GWEP), (ii) the outcome of any local test experiment is independent of the velocity of the (freely falling) apparatus, and (iii) the outcome of any local test experiment is independent of where and when in the universe it is performed. The GR is the unique gravitational theory which satisfies the SEP. In many gravitational theories appears the Nordtvedt effect about the Earth-Moon system, but in GR it does not appear. Indeed, this effect appears if the Earth and the Moon are accelerated toward the Sun with different accelerations. Any difference in those accelerations due to a failure of the distance Earth-Moon with the 29.53 days synodic period. The unexplained variation between the observation and the GR prediction of the distance Earth-Moon has amplitude of about 5.7 mm. This deviation from GR currently is considered as a violation of the SEP (i), which means that the gravitational and inertial masses are different. On the other hand, according to some recent results [13], the variation between the theoretical prediction and the observation has amplitude of about 0.28 mm. Notice that the standard deviation for the observed deviation of the distance Earth-Moon is about 4.1 mm. The mentioned paper [13] assumes that SEP (i) is true, but uses that the masses of the Earth and Moon are different, which is unimportant for the GR. Thus this explanation associates that SEP (i) is true, i.e. the gravitational and inertial masses are equal.

In section 3 we shall consider quite different experiment about the SEP. First we shall present in the next section the theory about the time dependent gravitational potential [9, 10].

In this paper we accept the well known metric from the general relativity, but shall not use it for equations of motion via a metric connection. It means that we accept the same explanation of the redshift and Shapiro time delay effects, just as the General Relativity. Moreover, these two experiments confirm the metric up to c^{-2}. The equations of motion via the new method were recently developed [11, 13, 12]. The other relativistic effects are explained in different ways. In this paper we shall use only the simple case when the metric (or the gravitational potential) depends on the time only. For this aim it is sufficient to know the metric components up to c^{-2}.

2. Time dependent gravitational potential

An idea about a linear (or almost linear) change of the gravitational potential in the universe was recently developed [9]. This change of the gravitational potential is apart from the change of the gravitational potential near the massive bodies. It is assumed that the sign of the gravitational potential V is such that the gravitational potential V is larger near the massive bodies. Under this choice of the sign of the gravitational potential V, if a light signal starts from a star with a frequency ν_0, then according to the general relativity after time t its frequency will be

$$\nu = \nu_0\Big(1 + \frac{t}{c^2}\frac{\partial V}{\partial t}\Big).$$

Specially, on a distance $R = ct$ its frequency will be

$$\nu = \nu_0\Big(1 + \frac{R}{c^3}\frac{\partial V}{\partial t}\Big).$$

On the other side, according to the Hubble law, this frequency is

$$\nu = \nu_0\Big(1 - \frac{RH}{c}\Big),$$

where H is the Hubble constant, $H \approx 60 - 70$ km/s/Mpc. According to the last two equalities, we obtain that

$$\frac{\partial V}{\partial t} = -c^2 H \approx -2 \times 10^3 \frac{\mathrm{cm}^2}{\mathrm{s}^3}. \tag{2.1}$$

Hence by accepting this linear change given by (2.1), the first and the most simple application is the explanation of the Hubble red shift. According to this, the distant galaxies do not move with enormous velocities, as it is currently accepted, but the main reason is the linear change of the gravitational potential, while the Doppler effect has a minor role. The reason of this change of the gravitational potential is probably caused by the change of the density of the dark energy. The following question is open: Is the value of H independent from the position and the time in the universe?

A question of great importance is how this effect influences the planetary orbits and orbital periods? This question is well studied [9, 10], and we give a brief view.

Let us denote by X, Y, Z, T the coordinates according to an observer where there is no time dependent gravitational potential. It is analogous as the gravitational theories use an observer far from the massive bodies, where the gravitation disappears. Since we assume that the time gravitational potential is present everywhere in the universe, such an observer practically does not exist, but however, we may adopt its existence. More precisely, dX, dY, dZ, and dT are the infinitesimal increments in the space-time coordinates at a considered point where there is a time dependent gravitational potential, according to the observer where the time dependent gravitational potential is absent. After norming these 1-forms according to the known metric from the GR we obtain the next 1-forms

$$w_x = \Big(1 + \frac{V}{c^2}\Big)^{-1} dX = \Big(1 + tH\Big) dX, \tag{2.2}$$

$$w_y = \Big(1 + \frac{V}{c^2}\Big)^{-1} dY = \Big(1 + tH\Big) dY, \tag{2.3}$$

$$w_z = \Big(1 + \frac{V}{c^2}\Big)^{-1} dZ = \Big(1 + tH\Big) dZ, \tag{2.4}$$

$$w_t = \Big(1 - \frac{V}{c^2}\Big)^{-1} dT = \Big(1 - tH\Big) dT, \tag{2.5}$$

Since the right sides of (2.2), (2.3), and (2.4) are not total differentials, the equations $dx = w_x$, $dy = w_y$, $dz = w_z$, and $dt = w_t$ are not solvable with respect to x, y, z, i.e. x, y, and z, are not functions of X, Y, Z, and T in general case. Only t is a function of T and we assume that $t = T = 0$ at a chosen moment. If we consider a chosen curve in the X, Y, Z, T space-time, then it corresponds to unique curve in the x, y, z, t space-time. Thus we agree to call x, y, z, and t "normed coordinates", although dx, dy, dz, and dt is only an orthonormal tetrade. Thus along a chosen curve, instead of (2.2-5) we can write

$$dx = \Big(1 + tH\Big) dX, \tag{2.2$'$}$$

$$dy = \Big(1 + tH\Big) dY, \tag{2.3$'$}$$

$$dz = \Big(1 + tH\Big) dZ, \tag{2.4$'$}$$

$$dt = \Big(1 - tH\Big) dT, \tag{2.5$'$}$$

and operate using the differential calculus. The equations (2.2$'$), (2.3$'$), and (2.4$'$) are not equalities between 1-form, but equalities along a chosen curve. The previously described model is still not well studied from the viewpoint of the differential geometry and it is an interesting subject for a future research.

Now the Hubble red shift is explained directly from (2.5$'$). It is sufficient to assume that (2.2$'$), (2.3$'$), (2.4$'$), and (2.5$'$) are satisfied, and then it is not necessary to speak about the time dependent gravitational potential. The coefficients $1 + tH$ and $1 - tH$ probably should be exponential functions of tH, but neglecting H^2 and smaller quantities we accept these linear functions. The coefficient $1 + tH$ can be replaced by $1 + TH$, neglecting the terms of order H^2.

From (2.2′), (2.3′), (2.4′), and (2.5′) we obtain

$$\Big(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\Big) = \Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big)(1 - 2tH) \tag{2.6}$$

and by differentiating this equality by T we have

$$\Big(\frac{d^2X}{dT^2}, \frac{d^2Y}{dT^2}, \frac{d^2Z}{dT^2}\Big) =$$

$$\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - 3tH\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - 2H\Big(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\Big) =$$

$$\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - 3tH\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - 2H\Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big). \tag{2.7}$$

Since we neglect the attraction forces and assume that there is no angular velocity, we assume axiomatically that in normed coordinates x, y, z, t there is no acceleration caused by the time dependent gravitational potential, i.e. we can put there $H = 0$. Thus, according to X, Y, Z, T coordinates there is an additional slight acceleration

$$-3tH\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - 2\Big(H\frac{dX}{dT}, H\frac{dY}{dT}, H\frac{dZ}{dT}\Big).$$

Since x, y, z, t are not functions of X, Y, Z, T, in order to see the influence of the constant H to the planetary orbits, we are looking for a functional dependence of the form

$$x = (1 + \lambda tH)\bar{X},\ y = (1 + \lambda tH)\bar{Y},$$

$$z = (1 + \lambda tH)\bar{Z},\ dt = (1 - \mu tH)d\bar{T}, \tag{2.8}$$

$\lambda = const.$ and $\mu = const.$, which yields to the same equality (2.7). Namely, we want to simulate the equation (2.7) via change of coordinates classically. As a consequence from (2.8) it is

$$\Big(\frac{d^2\bar{X}}{d\bar{T}^2}, \frac{d^2\bar{Y}}{d\bar{T}^2}, \frac{d^2\bar{Z}}{d\bar{T}^2}\Big) =$$

$$(1 - (\lambda + 2\mu)TH)\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - (2\lambda + \mu)H\Big(\frac{d\bar{X}}{d\bar{T}}, \frac{d\bar{Y}}{d\bar{T}}, \frac{d\bar{Z}}{d\bar{T}}\Big) =$$

$$(1 - (\lambda + 2\mu)TH)\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - (2\lambda + \mu)H\Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big). \tag{2.9}$$

Thus, comparing the right sides of (2.7) and (2.9) we obtain $\lambda = \frac{1}{3}$, $\mu = \frac{4}{3}$. According to (2.8) it is proved [9] that there is no perihelion precession caused by the time dependent gravitational potential. Moreover, neglecting the relativistic corrections of the planetary orbits, it is shown [9] that the planetary orbits are not axially symmetric and the angle from the perihelion to the aphelion is $\pi - \frac{\lambda H\Theta\sqrt{1-e^2}}{e\pi} = \pi - \frac{H\Theta\sqrt{1-e^2}}{3e\pi}$, while the angle from the aphelion to the perihelion is $\pi + \frac{\lambda H\Theta\sqrt{1-e^2}}{e\pi} = \pi + \frac{H\Theta\sqrt{1-e^2}}{3e\pi}$, where Θ is the orbital period and e is the eccentricity of the orbit. Notice that these angles are observed according to the observer where there is no time dependent gravitational potential. From (2.8) it follows that the quotient $\Theta_2 : \Theta_1$ of two consecutive orbital periods is equal to $1 + \mu\Theta H = 1 + \frac{4}{3}\Theta H$. This shows that each next orbit has a prolonged period for a factor $1 + \frac{4}{3}\Theta H$. But our time Θ is also prolonged for a factor $1 + \Theta H$ according to (2.5′). Thus we measure that each next orbit is prolonged for the factor

$$\Theta_2 : \Theta_1 = \frac{1 + \frac{4}{3}\Theta H}{1 + \Theta H} = 1 + \frac{1}{3}\Theta H. \tag{2.10}$$

Formula (2.10) can be applied also for the orbital period for arbitrary double stars [10]. From (2.10) we obtain

$$\dot{P}_b = \frac{1}{3} P_b H, \tag{2.11}$$

where P_b is the orbital period of the double stars. Formula (2.11) is tested for the binary pulsars B1885+09 [5] and B1534+12 [7, 8], which have very stable timings, and the results are satisfactory. Indeed, formula (2.11) together with the influence of decay of the orbital period caused by the gravitational radiation and a non-gravitational influence of kinematic nature in the galaxy, yield together to the measured value of $\dot{P}_b$ [9, 10]. Note that if we neglect the influence from the universe (2.11), then the change of the orbital period caused by the gravitational radiation would not fit with the experiments.

According to the previous discussion it also follows that the distance measured via laser equipment to any object moving freely on an orbit, for example the distance Earth-Moon, increases by the coefficient $1 + (1 - \lambda)HT = 1 + \frac{2}{3}HT$. The previous papers [9, 10] give explanation of the so called Pioneer anomaly, for the frequency shift in navigation of the spacecrafts Pioneer 10 and Pioneer 11 [1]. Notice that this deviation is explained [9, 10] to be basically induced by the change of the velocity (2.6) and the Doppler effect, but not from the acceleration (2.7). The increasing of the orbital period of the Moon, the distance to the Moon and the change of the average Earth's angular velocity are also considered [10]. These quantities depend on tidal dissipation and also on the time dependent gravitational potential. Including the influence from the time dependent gravitational potential, the discrepancies among the previous three changes become much smaller [10].

3. Is the SEP valid or not?

Now we are ready to consider the following experiment. Assume that there is a freely falling shielded laboratory toward the Earth and we consider a very short time interval when the shielded laboratory has velocity close to v toward the Earth. Then

$$-\frac{dV}{c^2dt} = \frac{GM}{R^2c^2}\frac{dR}{dt} = -\frac{GMv}{R^2c^2} = const.,$$

where R is the radius of the Earth, and M is its mass. Now the previous constant has the same role as the Hubble constant H in the previous section. Indeed, it is sufficient to take velocity $v = -2$ cm/s, i.e. 2 cm/s away from the Earth, and then the Earth's gravitational potential in the shielded laboratory changes just as in the universe.

Assume that in the shielded laboratory we have a gravitational body with small radius and a particle orbiting around it, such that the orbital plane is parallel to the ground. We assume that the orbital period $P_b = \Theta$ is much smaller than the considered time interval, while the velocity of the shielded laboratory is close to v. Now according to the results in the previous section, the quotient between two successive orbital periods observed far from gravitation (or from the Earth) is equal to $\Theta_2 : \Theta_1 = 1 - \frac{4}{3}\frac{GMvP_b}{R^2c^2}$. On the other hand, according to the same observer after time $P_b = \Theta$ the time seems to be slower for the quotient $1 - \frac{GMvP_b}{R^2c^2}$. Hence according to the observer from the shielded laboratory, the quotient between two successive orbital periods is observed to be

$$\Theta_2 : \Theta_1 = \frac{1 - \frac{4}{3}\frac{GMvP_b}{R^2c^2}}{1 - \frac{GMvP_b}{R^2c^2}} = 1 - \frac{1}{3}\frac{GMvP_b}{R^2c^2},$$

and hence

$$\dot{P}_b = -\frac{GMvP_b}{3R^2c^2}. \tag{3.1}$$

Notice that according to the observer far from gravitation (or from the Earth) appears an effect from the special relativity, such that both expressions $1 - \frac{4}{3}\frac{GMvP_b}{R^2c^2}$ and $1 - \frac{GMvP_b}{R^2c^2}$ should be multiplied by the same factor, but their quotient and (3.1) remain unchanged. The argument that $\dot{P}_b \neq 0$ is a result of the fact that x, y, and z are nonholonomy coordinates.

More generally, we shall show now that the change of the orbital period and the change of the orbital distance depend only on the PPN parameter γ, which has value 1. Indeed, instead of the equations $(2.2' - 5')$ we have the equations

$$dx = \Big(1 - \gamma t \frac{GMv}{R^2c^2}\Big) dX, \tag{3.2}$$

$$dy = \Big(1 - \gamma t \frac{GMv}{R^2c^2}\Big) dY, \tag{3.3}$$

$$dz = \Big(1 - \gamma t \frac{GMv}{R^2c^2}\Big) dZ, \tag{3.4}$$

$$dt = \Big(1 + k t \frac{GMv}{R^2c^2}\Big) dT, \tag{3.5}$$

where $k = const.$ and γ is the PPN parameter. We assume that $\frac{GMv}{R^2c^2}$ is extremely small, such that during a very short orbital period, the value of $\frac{GMv}{R^2c^2}$ is almost a constant. Now analogously to (2.6) and (2.7) we obtain

$$\Big(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\Big) = \Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big)\Big(1 + (k+\gamma) t \frac{GMv}{R^2c^2}\Big) \tag{3.6}$$

and

$$\Big(\frac{d^2X}{dT^2}, \frac{d^2Y}{dT^2}, \frac{d^2Z}{dT^2}\Big) = \Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) +$$

$$(2k+\gamma) t \frac{GMv}{R^2c^2}\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) + (k+\gamma)\frac{GMv}{R^2c^2}\Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big). \tag{3.7}$$

Analogously to (2.8) we consider an adopted transformation

$$x = \Big(1 - \lambda t \frac{GMv}{R^2c^2}\Big)\bar{X}, \ y = \Big(1 - \lambda t \frac{GMv}{R^2c^2}\Big)\bar{Y},$$

$$z = \Big(1 - \lambda t \frac{GMv}{R^2c^2}\Big)\bar{Z}, \ dt = \Big(1 + \mu t \frac{GMv}{R^2c^2}\Big) d\bar{T}, \tag{3.8}$$

where $\lambda = const.$ and $\mu = const.$ From (3.8) we obtain

$$\Big(\frac{d^2\bar{X}}{d\bar{T}^2}, \frac{d^2\bar{Y}}{d\bar{T}^2}, \frac{d^2\bar{Z}}{d\bar{T}^2}\Big) = \Big(1 + (\lambda + 2\mu) T \frac{GMv}{R^2c^2}\Big)\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) +$$

$$(2\lambda + \mu)\frac{GMv}{R^2c^2}\Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big). \tag{3.9}$$

Hence, comparing (3.7) and (3.9) we obtain the system

$$2k+\gamma=\lambda+2\mu, \qquad k+\gamma=2\lambda+\mu,$$

whose solution is $\lambda=\frac{\gamma}{3}$ and $\mu=k+\frac{\gamma}{3}$. Finally, analogous to (2.10) and (2.11) we obtain

$$\Theta_2:\Theta_1=\frac{1-(k+\frac{\gamma}{3})\Theta\frac{GMv}{R^2c^2}}{1-k\Theta\frac{GMv}{R^2c^2}}=1-\frac{\gamma}{3}\Theta\frac{GMv}{R^2c^2}, \tag{3.10}$$

$$\dot{P}_b=-\frac{\gamma}{3}P_b\frac{GMv}{R^2c^2}, \tag{3.11}$$

and the distance r between two bodies measured from the shielded laboratory changes according to the formula

$$\delta r=-r(\gamma-\lambda)T\frac{GMv}{R^2c^2}=-\frac{2\gamma}{3}rT\frac{GMv}{R^2c^2}. \tag{3.12}$$

Notice that both gravitational effects $\dot{P}_b$ and δr depend only on γ, but do not depend on the coefficient k.

On the other hand, according to the SEP (ii) the considered gravitational experiment (measuring the quotient between two successive orbital periods according to the observer from the shielded laboratory) should not depend on the velocity v of the shielded laboratory, and hence

$$\dot{P}_b=0. \tag{3.13}$$

Hence the SEP is not valid. But notice that the SEP is deduced in GR which considers only a set of coordinate systems that cover 4-dimensional space-time manifold, with assumption of functional dependence between any two coordinate systems with non-empty intersection. This viewpoint for dealing with coordinate systems do not correspond to their appearances in the physical reality. Since the formula (3.1) in case of binary pulsars is confirmed (section 2), we conclude that it is necessary to consider differential 1-forms (2.2-5) instead of functional dependencies. Indeed, according to the GR the transformation of the coordinates of the observer far from gravitation and to the coordinates to the observer in the shielded laboratory is given by the functional dependencies

$$x=X\Big(1-\frac{GM}{c^2(R-\frac{1}{2}\frac{GM}{R^2}t^2)}\Big),$$

$$y=Y\Big(1-\frac{GM}{c^2(R-\frac{1}{2}\frac{GM}{R^2}t^2)}\Big),$$

$$z = Z\Big(1 - \frac{GM}{c^2(R - \frac{1}{2}\frac{GM}{R^2}t^2)}\Big),$$

$$dt = dT\Big(1 + \frac{GM}{c^2(R - \frac{1}{2}\frac{GM}{R^2}t^2)}\Big).$$

The last equality can be integrated and written in the form $t = f(T)$. Now according to the coordinates X, Y, Z, T due to the observer in the shielded laboratory $\dot{P}_b = 0$, and this is the reason that according to GR the SEP is valid, although we saw previously that SEP (ii) is not valid.

At the end we explain how the previous problem with the coordinate systems in GR can be overcome at the first post-Newtonian approximation (1PN), in case of gravitation of a spherical body with mass M. First let us consider a normalization of the 1-forms dX, dY, dZ, and dT. Analogously to $(2.2' - 5')$ we have

$$dx = \mu^{-1}dX, \tag{3.14a}$$

$$dy = \mu^{-1}dY, \tag{3.14b}$$

$$dz = \mu^{-1}dZ, \tag{3.14c}$$

$$dt = \mu dT, \tag{3.14d}$$

where $\mu = 1 + \frac{GM}{rc^2}$. A direct consequence of the previous equations yields to

$$\Big(\frac{d^2X}{dT^2}, \frac{d^2Y}{dT^2}, \frac{d^2Z}{dT^2}\Big) = \frac{2}{\mu}\frac{d\mu}{dT}\Big(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\Big) + 3\mu^2\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big). \tag{3.15}$$

The problem about motion of a particle with a zero mass will be solved after determination of the acceleration $\left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\right)$ in normed coordinates. This acceleration can be obtained via a connection [11, 12, 13], which is nonlinear, and preserves the Minkowskian flat metric. It depends on the fields of 3-vector of acceleration (a_x, a_y, a_z) and angular velocity (w_x, w_y, w_z), and do not depend on the nature of the source of these 3-vector fields. Indeed, it is irrelevant whether (a_x, a_y, a_z) is caused by gravitation or it is a centrifugal acceleration or any other inertial force. Using this connection [12] are obtained the same results as in GR for the deflection of the light near the Sun, the perihelion (periastron) shift of a planetary orbit (binary pulsar's orbit), the frame-dragging effect of inertia and the geodetic precession. Thus these four effects should be considered as quasi-gravitational effects, and they are closer to the Special Relativity. On the other side, the effects like the gravitational red shift, Shapiro time delay, the gravitational radiation and the results from section 2 are purely gravitational because for their derivation are used the equations (3.14) or (3.15) or the curvature tensors in case of gravitational radiation.

4. An anomaly in the radial motion of a particle in a spherical gravitational field

While in the previous section we considered an anomaly in the GR caused by consideration of the functional dependence between different coordinates, in this section the analogous anomaly will appear by using the metric connection for the geodesic lines in the curved space-time.

If a particle moves with a large velocity radially in the gravitational field, then the functional dependence between its velocity and the distance to the center of the Earth is given by the following formula [6]

$$\frac{\sqrt{1-\frac{2GM}{rc^2}}}{\sqrt{1-\frac{v^2}{c^2}}} = C \quad (= const.), \tag{4.1}$$

where $v = \frac{dr}{dt}\left(1-\frac{2GM}{rc^2}\right)^{-1}$. Multiplying this equality with m_0c^2, we obtain the energy in the following form

$$E_{kin}\sqrt{1-\frac{v^2}{c^2}} = const.$$

but not in the classical form $E_{kin} + E_{pot} = const.$ If we consider a weak gravitational field, i.e. $\frac{GM}{rc^2} << 1$, then (4.1) can be written as

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = C\frac{GM}{rc^2} + C,$$

and by differentiating

$$\frac{d}{dt}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = C\frac{GM}{c^2}\frac{d}{dt}\frac{1}{r},$$

$$\frac{d}{dt}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = C\frac{a}{c^2}\frac{dr}{dt}, \tag{4.2}$$

where $a = -\frac{GM}{r^2}$ is the acceleration toward the Earth.

In short time interval the acceleration a can be considered to be almost a constant, and it is natural to expect according to the EP that the motion given by (4.1), i.e. (4.2) is just the same as a motion in Special Relativity under a constant acceleration a. Such motion depends only on the constant acceleration a and the initial value of v. But, on the other hand, the equation (4.2) depends on the constant C which is related to the motion in the gravitational field, and it has no role for the Special Relativity motion under a constant force. This anomaly of the GR equations of motion show that they are not close to the special relativity as it is natural to expect. Finally notice that the analogous equations presented in the mentioned paper [12] do not have such anomaly considered in normed coordinates.

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