

50 YEARS SETS WITH POSITIVE REACH - A SURVEY -

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Abstract. The purpose of this paper is to summarize results on various aspects of sets with positive reach, which are up to now not available in such a compact form. After recalling briefly the results before 1959, sets with positive reach and their associated curvature measures are introduced. We develop an integral and current representation of these curvature measures and show how the current representation helps to prove integralgeometric formulas, such as the principal kinematic formula. Also random sets with positive reach and random mosaics (or the more general random cell-complexes) with general cell shape are considered.

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