# PROBING THE GRAVITATIONAL UNIVERSE WITH GRAVITATIONAL WAVES


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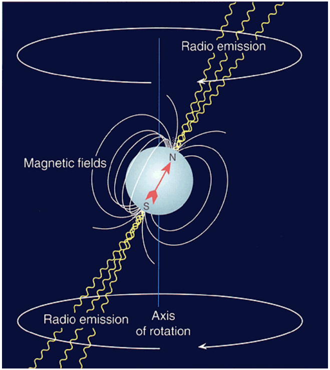
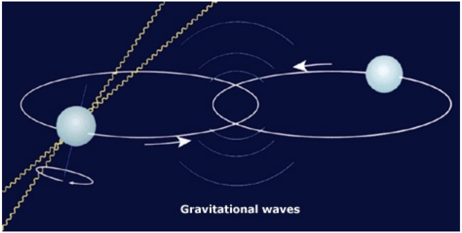
# The binary pulsar PSR 1913+16

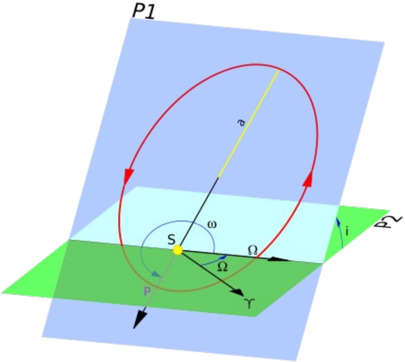



- The pulsar PSR 1913+16 is a rapidly rotating neutron star emitting radio waves like a lighthouse toward the Earth.
  Non-orbital parameters
  1. $P_{\text{pulsar}} = 59\,\mathrm{ms}$ pulsar period
  2. $\dot{P}_{\text{pulsar}} < 10^{-12}$ pulsar spin-down
- This pulsar moves on a (quasi-)Keplerian close orbit around an unseen companion, probably another neutron star

# The Keplerian orbit of the binary pulsar

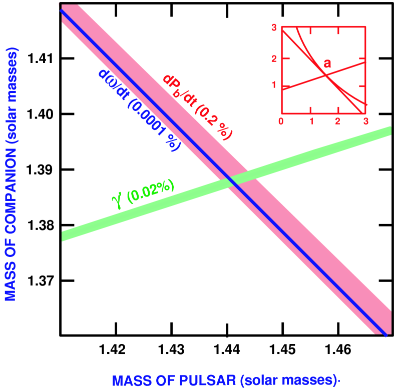
The analysis of the arrival time of successive pulsar's radio pulses yields an accurate determination of the Keplerian orbit




Keplerian parameters

1. $a\,\sin i = 700\,000\,\text{km}$ projected semi-major axis
2. $e = 0.617$ eccentricity
3. $P = 7.75\,\text{h}$ orbital period

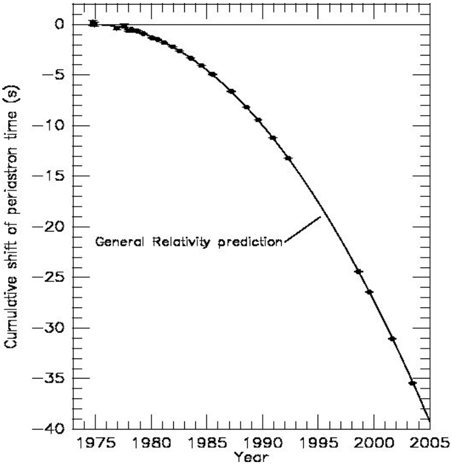
# Measurement of general relativistic effects




Post-Keplerian parameters

1. $\dot{\omega} = 4.2\,°/\text{yr}$ relativistic advance of periastron
2. $\gamma = 4.3\,\text{ms}$ gravitational red-shift and second-order Doppler effect
3. $\dot{P} = -2.4\,10^{-12}\text{s/s}$ secular decrease of orbital period

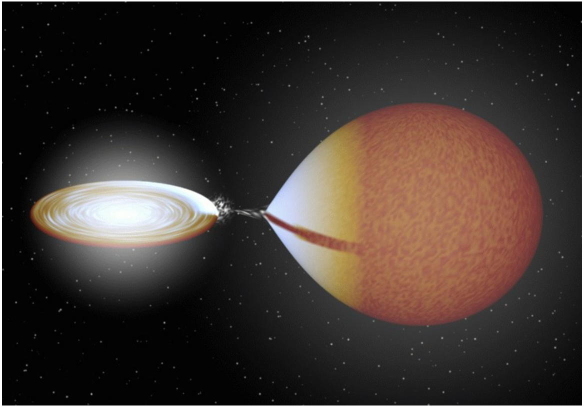
# The orbital decay of binary pulsar




## Prediction from general relativity theory

$$\dot{P} = -\frac{192\pi}{5c^5}\,\frac{\mu}{M}\left(\frac{2\pi G\,M}{P}\right)^{5/3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}} \approx -2.4\,10^{-12}$$

# Cataclysmic variables



- An evolved normal star — the Secondary, with mass $M_2$ — fills its Roche lobe and transfers mass to a more massive companion — the Primary, with mass $M_1 > M_2$ — which is a white dwarf
- An accretion disk of heated matter forms around the Primary and UV and X rays are emitted because of the high temperature

# Loss of angular momentum in cataclysmic variables

1. The orbital angular momentum is $J = GM_1 M_2 (a/GM)^{1/2}$ so we deduce

$$\frac{\dot{a}}{a} = \frac{2\dot{J}}{J} + \frac{2(-\dot{M}_2)}{M_2}\left(1 - \frac{M_2}{M_1}\right)$$

   where $-\dot{M}_2$ is the mass transfer from $M_2$ to $M_1$

2. The mass transfer tends to increase the distance $a$ between the two stars (since $M_2 < M_1$) so to extlain the long-lived cataclysmic binaries we need a mechanism of loss of angular momentum

3. When $P \lesssim 2\,\text{hours}$ there is only one mechanism: gravitational radiation

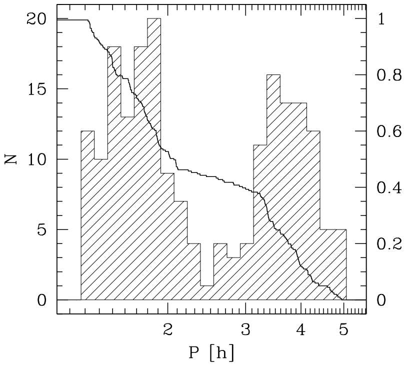
$$\left(\frac{\dot{J}}{J}\right)^{\text{GW}} = -\frac{32G^2}{5c^5}\frac{M_1 M_2}{a^4}$$

4. With $\dot{a} = 0$ we get an estimate for $-\dot{M}_2$ and the result is in good agreement with the mass tranfer infered from X-ray observations

# Histogram of cataclysmic variables

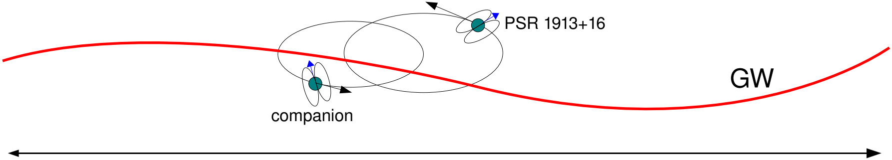
The presence of this peak (corresponding to orbital periods $P \lesssim 2$ hours) is only explained by gravitational radiation

$\Downarrow$



# What is a gravitational wave?

- A gravitational wave (GW) is a ripple in the curvature of space-time propagating at the speed of light
- GWs are generated by the dynamics and orbital motion of the source
- They are more like sound waves rather than light waves



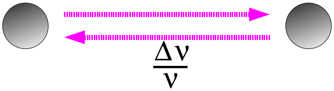

Wavelength of gravitational radiation $\lambda = c\,P = 50$ AU for binary pulsar

# Einstein's equivalence principle

- In the neighbourhood of any event $\mathcal{P}$ in space-time one can construct locally inertial coordinates $\{X^\alpha\}$ such that
  1. the laws of special relativity are valid at $\mathcal{P}$
  2. the deviation from special relativity around $\mathcal{P}$ is of second order

$$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} \underbrace{R_{\alpha\mu\beta\nu}}_{\text{curvature tensor}} X^\mu X^\nu + \mathcal{O}\left(X^3\right)$$

- Consequence for GW detection: only modification of relative distances can be measured, for instance by measuring the frequency shift of light exchanged between two masses



# Einstein's field equations

1. They are based on the Einstein-Hilbert Lagrangian

$$L = \underbrace{\frac{c^4}{16\pi G}\sqrt{-g}\,R}_{\text{gravitational field}} + \underbrace{L_\phi[g,\phi]}_{\text{matter fields}}$$

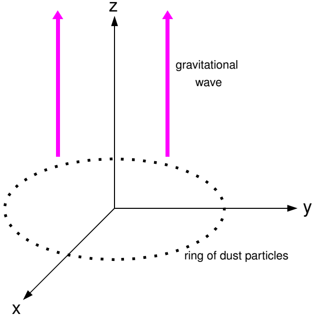
2. System of ten differential equations of second order for the ten metric coefficients $g_{\mu\nu}(x^\rho)$

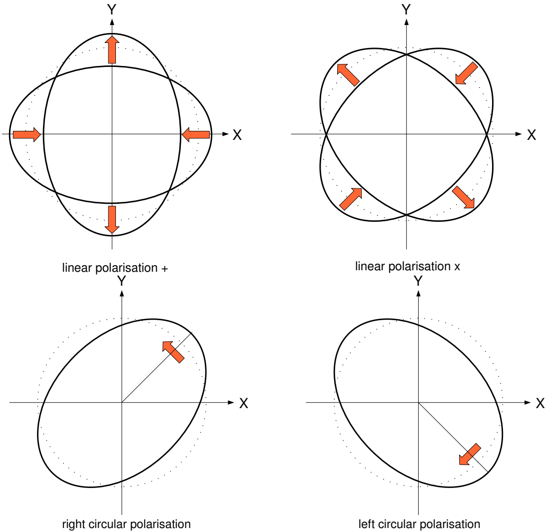
$$\underbrace{G^{\mu\nu}[g,\partial g,\partial^2 g]}_{\text{Einstein tensor}} + \underbrace{\Lambda\, g^{\mu\nu}}_{\text{cosmological term}} = \frac{8\pi G}{c^4}\underbrace{T^{\mu\nu}[g,\phi]}_{\text{energy-momentum tensor}}$$

3. The field equations imply, by the contracted Bianchi and Ricci identities, the equations of motion of matter

$$\left.\begin{aligned}\nabla_\nu G^{\mu\nu} &\equiv 0\\ \nabla_\nu g^{\mu\nu} &\equiv 0\end{aligned}\right\} \Longrightarrow \nabla_\nu T^{\mu\nu} = 0$$

# Modes of gravitational waves



# Gravitational waves for the Mathematician

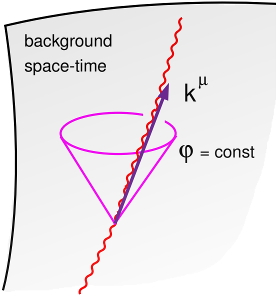
1. Propagation of some on/off signal from a source to a detector means that the theory should admit solutions with discontinuities
2. Metric of space-time is
   - everywhere $C^1$
   - piecewise $C^3$
3. It allows for surfaces of discontinuities ("characteristic surfaces") which correspond to wavefronts of pure GW propagating at the speed of light

# Gravitational waves for the WKB Physicist

A small deformation of a given background space-time

$$g_{\mu\nu} = \underbrace{g^0_{\mu\nu}}_{\text{background metric}} + \underbrace{h_{\mu\nu}\, e^{\mathrm{i}\,\omega\,\varphi(x)}}_{\text{perturbation}}$$

in the high-frequency limit $\omega \longrightarrow +\infty$




1. The wave vector $k_\mu = \partial_\mu \varphi$ is null

$$k^\mu k_\mu = 0$$

2. The motion is geodesic

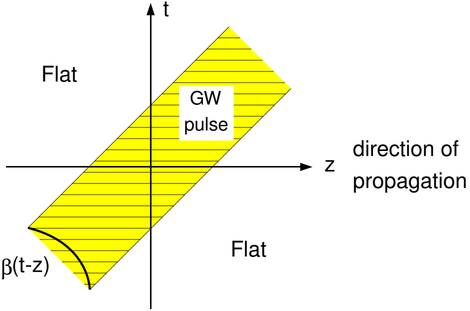
$$k^\nu \nabla_\nu k_\mu = 0$$

# Gravitational waves for the General-Relativity Purist

A plane wave which is an exact solution of Einstein's field equations

$$ds^2 = -dt^2 + L^2 \left(e^{2\beta}dx^2 + e^{-2\beta}dy^2\right) + dz^2$$

where $\beta = \beta(t-z)$ is the wave profile and $L = L(t-z)$ is the background factor determined from the field equations



# Gravitational waves for the Astrophysicist

1. The GW amplitude is given by the first quadrupole formula

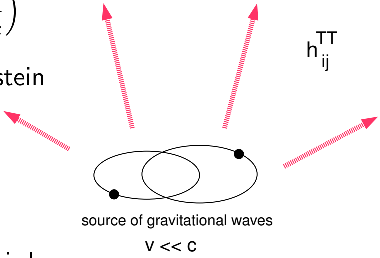
$$h^{\mathrm{TT}}_{ij}(\mathbf{x},t) = \frac{2G}{c^4 r}\, P_{ijkl}(\mathbf{n})\, \frac{d^2 Q_{kl}}{dt^2}\left(t - \frac{r}{c}\right)$$

2. The total GW energy flux is given by the Einstein quadrupole formula

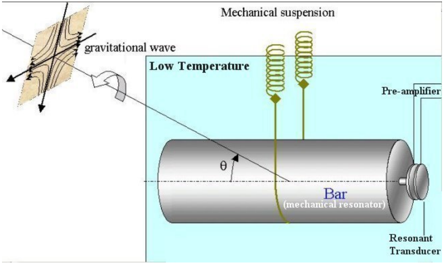
$$\left(\frac{dE}{dt}\right)^{\mathrm{GW}} = \frac{G}{5c^5}\,\frac{d^3 Q_{ij}}{dt^3}\,\frac{d^3 Q_{ij}}{dt^3}$$

3. The radiation reaction force is given by the third quadrupole formula

$$\mathcal{F}^{\mathrm{RR}}_i = \frac{2G}{5c^5}\,\rho\, x^j\,\frac{d^5 Q_{ij}}{dt^5}$$



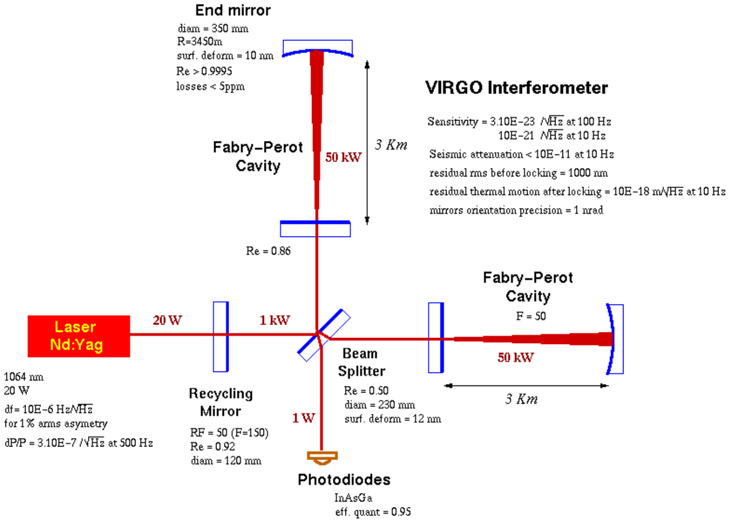
# Weber bars for the detection of gravitational waves




The mechanical vibrations are amplified when the gravitational wave frequency $\omega$ happens to be close to the bar's fundamental frequency $\Omega$

$$\delta\ddot{L} + \Omega^2 \delta L = \frac{L}{2}\ddot{h} \quad \Longrightarrow \quad \delta L_0 = \frac{L h_0}{2} \frac{\omega^2}{\omega^2 - \Omega^2}$$

# Principle of the laser interferometric GW detector



# Detecting a very weak GW signal

- The displacement of the end mirrors relative to the beam splitter is

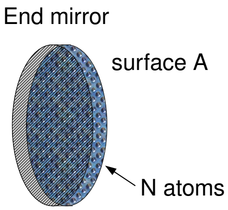
$$\frac{\delta L}{L} \sim \frac{h}{2}$$

with $L = 3\,\mathrm{km}$ for VIRGO.

- For binary systems at a distance $\sim 100\,\mathrm{Mpc}$ we have $h \sim 10^{-23}$

$$\delta L \sim 10^{-20}\,\mathrm{m} = 10^{-5}\,\mathrm{fermi}\,!$$

How is it possible to detect such a tiny displacement?




- One measures the collective displacement of $N$ atoms forming an atomic layer on the surface $A$ of the mirror

$$N \sim 10^{18} \quad \Longrightarrow \quad \delta L_{\mathrm{eff}} \sim \sqrt{N}\,\delta L \sim 10^{-11}\,\mathrm{m} = 0.1\,\text{Å}$$

which is of the order of inter-atomic distances

# Ground-based laser interferometric detectors

LIGO

GEO

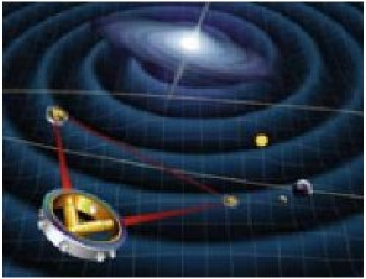
VIRGO

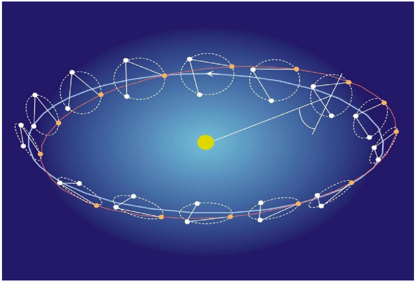
LIGO/VIRGO/GEO observe the GWs in the high-frequency band

$$10\,\mathrm{Hz} \lesssim f \lesssim 10^3\,\mathrm{Hz}$$

# Space-based laser interferometric detector

LISA

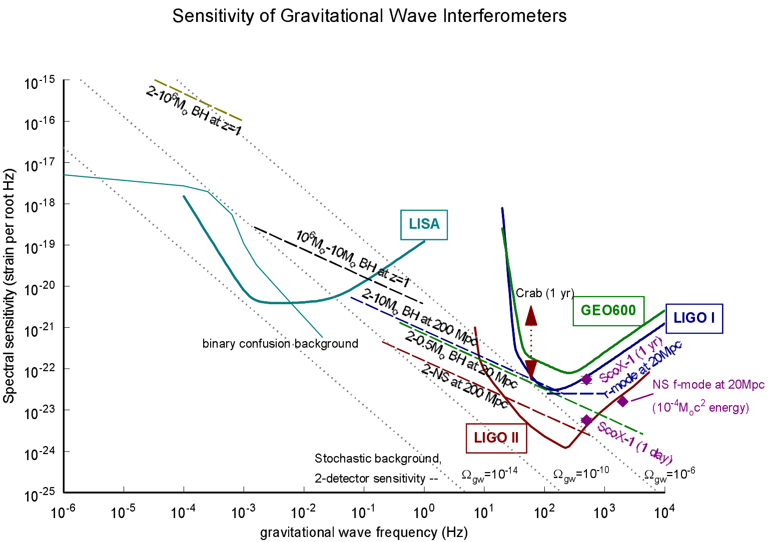




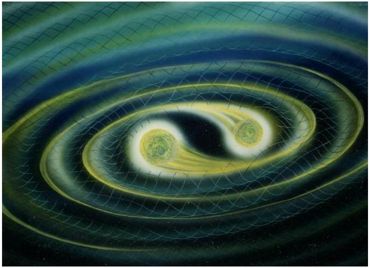
LISA will observe the GWs in the low-frequency band

$$10^{-4}\,\mathrm{Hz} \lesssim f \lesssim 10^{-1}\,\mathrm{Hz}$$

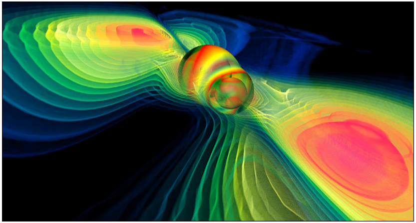
# GW sources for LISA and LIGO/VIRGO



# The inspiral and merger of compact binaries



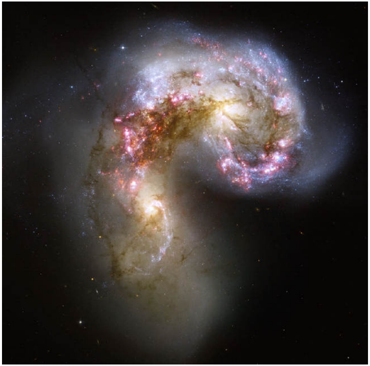
Neutron stars spiral and coalesce



Black holes spiral and coalesce

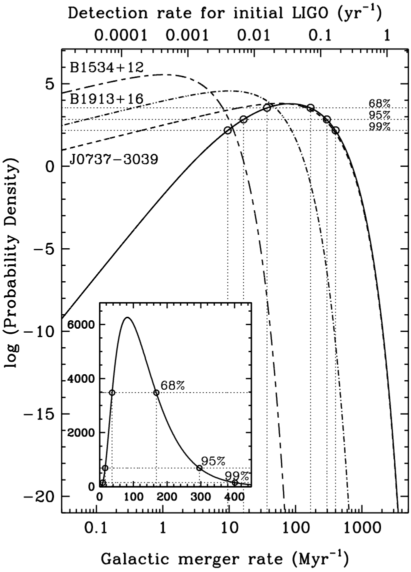
1. Neutron star ($M = 1.4\, M_\odot$) events will be detected by ground-based detectors LIGO/VIRGO/GEO
2. Stellar size black hole ($5\, M_\odot \lesssim M \lesssim 20\, M_\odot$) events will also be detected by ground-based detectors
3. Supermassive black hole ($10^5\, M_\odot \lesssim M \lesssim 10^8\, M_\odot$) events will be detected by the space-based detector LISA

# Supermassive black-hole coalescences as detected by LISA

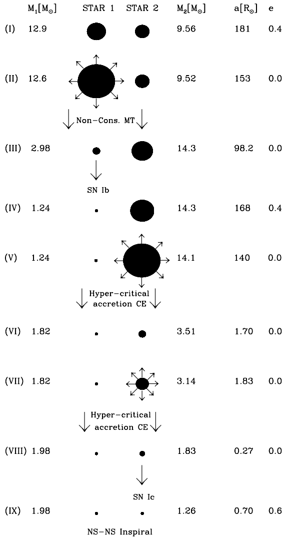


When two galaxies collide their central supermassive black holes may form a bound binary system which will spiral and coalesce. LISA will be able to detect the gravitational waves emitted by such enormous events anywhere in the Universe

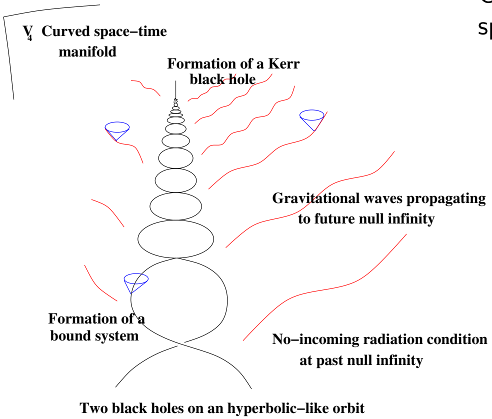
# Estimate of the number of double neutron star binaries



# Formation of double neutron star binaries



# The two-body problem in General Relativity

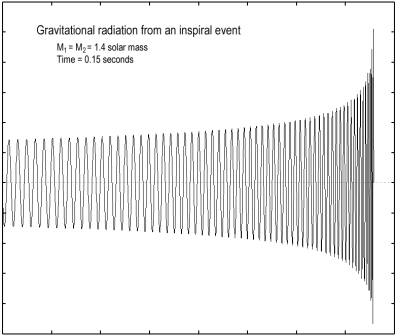



The solution of the two-body problem in General Relativity would consist of a space-time manifold describing

1. Two black holes on an initial hyperbolic-like (scattering) orbit
2. The formation of a bounded binary system by emission of gravitational radiation
3. The long inspiral phase where the black holes gradually come close to each other
4. The detailed process of merger of the two black hole horizons
5. The emission of quasi-normal mode radiation by the final object untill the formation of a stationary (Kerr) black hole

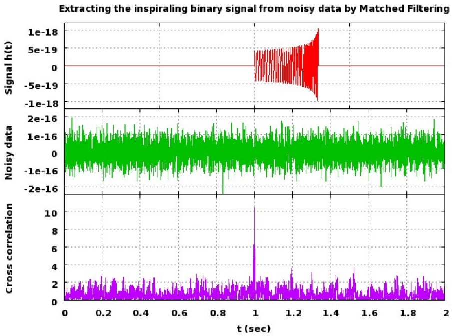
# The inspiral (or chirp) of compact binary systems

- The most interesting known source of gravitational waves for the LIGO/VIRGO detectors and a very important one for LISA
- The dynamics of these systems is driven by gravitational radiation reaction effects or equivalently by the loss of energy by gravitational radiation
- Theoretical waveforms (templates) for detection and analysis of the signals should be very accurate in terms of a post-Newtonian expansion
- Post-Newtonian waveforms for the inspiral should be completed by numerical calculations of the merger and ringdown phases



# Matched filtering of the chirp signal

In the matched filtering technique, one cross correlates the noisy output of a detector with theoretically computed waveforms or templates




Templates must remain in phase with the exact waveform as long as possible. If the signal and template lose phase with each other their cross-correlation will be significantly reduced and one may lose the event altogether

# Approximation scheme in general relativity

- The dominant radiation reaction effects appears at order $(v/c)^5$ beyond the Newtonian force, where $v$ is the typical velocity in the source and $c$ is the speed of light

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \underbrace{\mathcal{F}_\mathrm{N}}_{\text{Newtonian force}} + \cdots + \underbrace{\frac{1}{c^5}\mathcal{F}_\mathrm{RR}}_{\text{Radiation reaction}} + \cdots$$

  At leading order the RR force yields the quadrupole formula for the emission of gravitational radiation

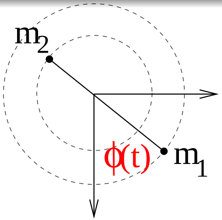
- During the inspiral the dynamics is adiabatic

$$T_\mathrm{RR} \gg T_\mathrm{orbital}$$

  with adiabatic parameter which is small in a post-Newtonian (PN) sense

$$\frac{\dot{\omega}}{\omega^2} = \mathcal{O}\left[\left(\frac{v}{c}\right)^5\right]$$

# Inspiralling compact binaries: the PN theorist's paradise




The orbital phase $\phi(t)$ should be monitored in LIGO/VIRGO detectors with precision

$$\delta\phi \sim \pi$$

$$\phi(t) = \underbrace{\phi_0 - \frac{1}{32\nu}\left(\frac{GM\omega}{c^3}\right)^{-5/3}}_{\text{result of the quadrupole formalism (sufficient for the binary pulsar)}} \Bigg\{ 1 + \underbrace{\frac{1\text{PN}}{c^2} + \frac{1.5\text{PN}}{c^3} + \cdots + \frac{3\text{PN}}{c^6} + \cdots}_{\text{needs to be computed with high PN precision}} \Bigg\}$$

Detailed data analysis (using the sensitivity noise curve of LIGO/VIRGO detectors) show that the required precision is at least 2PN for detection and 3PN for parameter estimation

# PN orbital phase of inspiralling compact binaries

The PN expansion of the orbital phase is obtained as

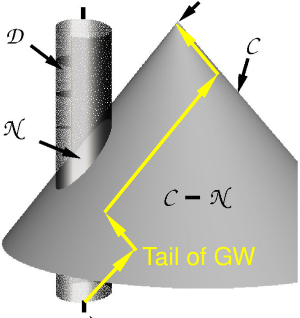
$$\begin{aligned}\phi(\omega) &= \phi_0 - \frac{1}{32\nu}\left(\frac{GM\omega}{c^3}\right)^{-5/3}\Bigg\{1 \\ &\quad + \overbrace{\left(\frac{3715}{1008}+\frac{55}{12}\nu\right)\left(\frac{GM\omega}{c^3}\right)^{2/3}}^{\text{1PN}} \\ &\quad \overbrace{-10\pi\left(\frac{GM\omega}{c^3}\right)}^{\text{1.5PN (tail)}} \\ &\quad + \overbrace{\left(\frac{15293365}{1016064}+\frac{27145}{1008}\nu+\frac{3085}{144}\nu^2\right)\left(\frac{GM\omega}{c^3}\right)^{4/3}}^{\text{2PN}} + \cdots\Bigg\}\end{aligned}$$

These strange-looking coefficients are reviewed in the Living Review in Relativity

# Typical coefficient in the 3PN orbital phase

$$\frac{12348611926451}{18776862720}$$

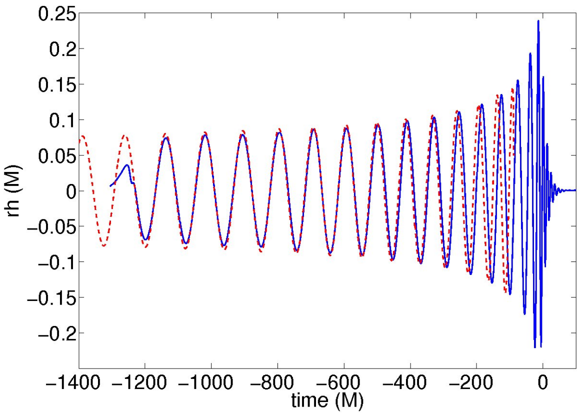
# Tails are an important part of the signal




Tails are produced by backscatter of GWs on the curvature induced by the matter source's total mass $M$

$$\delta h_{ij}^{\text{tail}} = \frac{4G}{c^4 D} \underbrace{\frac{G M}{c^3} \int_{-\infty}^{t} \mathrm{d}t' Q_{ij}(t') \ln\left(\frac{t-t'}{\tau_0}\right)}_{\text{The tail is dominantly a 1.5PN effect}} + \cdots$$

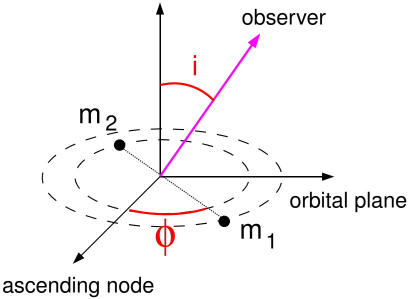
# Matching the PN inspiral to numerical merger waveforms




The numerical merger waveform is matched with high accuracy to the PN inspiral waveform. Current precision of the PN waveform is

1. 3.5PN order in phase
2. 3PN order in amplitude

# Compact binary systems are standard GW sirens



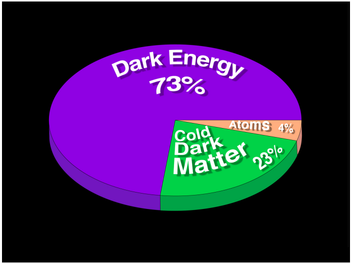

Polarisation states of GW from a compact binary system

$$h_+ = \frac{2G\mu}{c^2 D_\mathrm{L}} \left(\frac{GM\omega}{c^3}\right)^{2/3} \left(1+\cos^2 i\right) \cos\left(2\phi\right)$$

$$h_\times = \frac{2G\mu}{c^2 D_\mathrm{L}} \left(\frac{GM\omega}{c^3}\right)^{2/3} \left(2\cos i\right) \sin\left(2\phi\right)$$

The distance of the source $D_\mathrm{L}$ is measurable from the GW signal

# Supermassive black-hole binaries as dark energy probes




Supermassive black-hole coalescences will be observed by LISA up to high red-shift $z$. In the concordance model of cosmology the distance $D_\mathsf{L}$ is

$$D_\mathsf{L}(z) = \frac{1+z}{H_0} \int_0^z \frac{\mathrm{d}z'}{\sqrt{\Omega_\mathsf{M}(1+z')^3 + \Omega_\mathsf{DE}(1+z')^{3(1+w)}}}$$

LISA will be able to constrain the equation of state of dark energy $w = p_\mathsf{DE}/\rho_\mathsf{DE}$ to within a few percent