The Many Faces of Black Holes

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Reference: Living Reviews, AA & Badri Krishnan

'Newer' results:

Baez, Beetle, Booth, Corichi, Engle, Gourgoulhon, Fairhurst, Jaramillo, Krasnov, Krishnan, Lewadowski, Liko, Pawlowski, Taveras, Van Den Broeck, Varadarajan, ...

Living Reviews Colloquium, Berlin, June 3rd 2008

Dedication

Respectfully dedicated to the memory of

Jürgen Ehlers

The Founding Director of the
Max Planck Institut für Gravitationsphysik
The Albert Einstein Institut

A brilliant mind, an impeccable scientist.

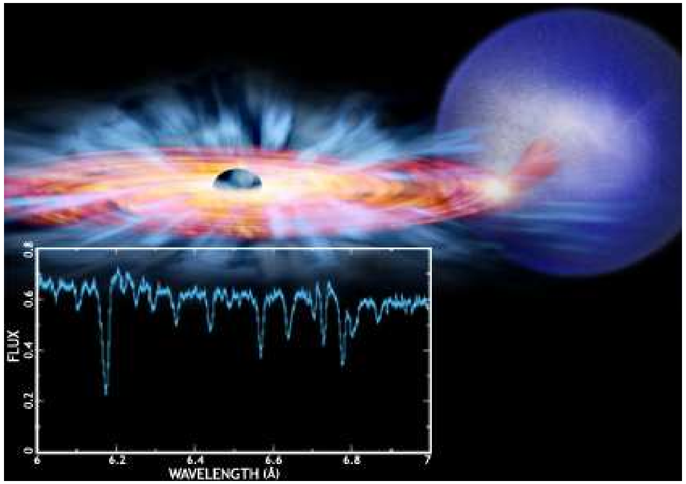
Teacher, Mentor and Friend

• Black holes are widely recognized as the engines that drive the most energetic phenomena in astrophysics. But they have also been the engines behind some of the most unexpected and fascinating advances in fundamental physics over the last three decades.

• Goal of the Talk: An overview of the profound impact black holes have had on the conceptual fabric of general relativity, quantum theory and statistical mechanics; why they continue to be fascinating, intriguing and vexing.

Organization:

1. Historical & Conceptual Setting
2. Black Hole Horizons
3. Black Hole Entropy
4. Information loss
5. Summary and Discussion



1. Historical and Conceptual Setting

• Escape Velocity: $V_e = \sqrt{\frac{2GM}{R}}$

If $V_e \geq c \;\Leftrightarrow\; \frac{2GM}{Rc^2} \geq 1$: light will not escape from the surface of the body: Black Hole!

• Suppose for simplicity the body has uniform density. Then,
$M = \frac{4\pi}{3} R^3 \rho \;\Rightarrow$ Black hole iff $\frac{8\pi}{3} \frac{G}{c^2} \rho R^2 \geq 1$.

Two ways of achieving this:
⋆ ρ large; Say $\rho \sim 6 \times 10^{16} \mathrm{gm/cm^3}$; then can form a BH or radius $\sim 1\mathrm{km}$.
[Recall, Nuclear density $\sim 10^{14} \mathrm{gm/cm^3}$.]
⋆ ρ small, say density of water. Then if $R \sim 2.5 \times 10^8 \mathrm{km}$, again we have a black hole!

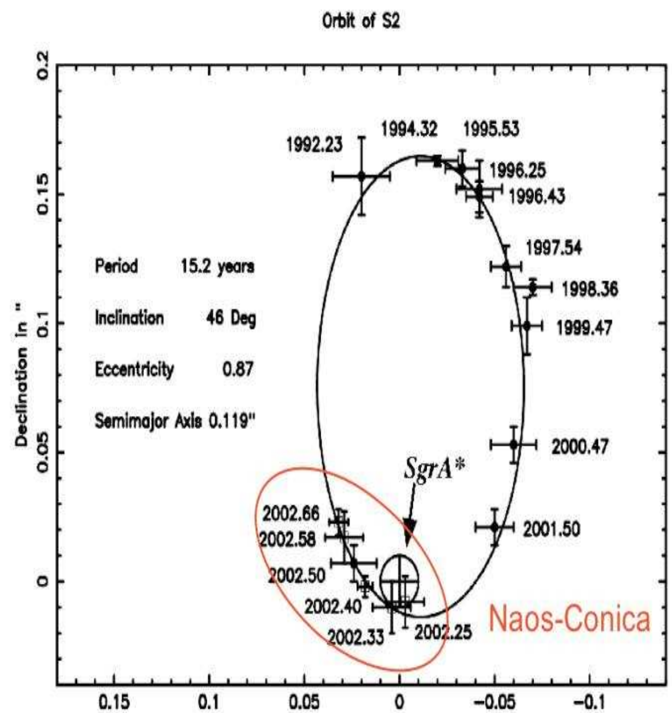
• Interestingly, Nature uses both these avenues. First type of black holes result from stellar collapse; few km in size; a few times $M_\odot$. The second exist in galactic centers; 10^6 to 10^9 $M_\odot$.

These ideas are very old. Surge of interest in the late 1700's:

If there should exist in nature any [such] bodies .... we could have no information from sight; yet if any other luminous bodies should happen to revolve around them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis.

John Mitchell

Phil. Trans. R. Soc. (Lon) (1784)



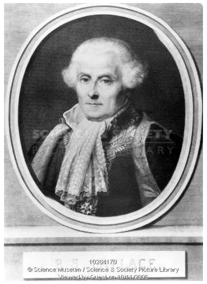
These ideas are very old. Surge of interest in the late 1700's:

A luminous body of the same density as earth, whose diameter is 250 times larger than that of the sun, can by its attractive power prevent its light rays from reaching us, and consequently, largest bodies in the universe could remain invisible to us.

... there exist, in the immensity of space, opaque bodies as considerable in magnitude, and perhaps equally as numerous as stars.

M Le Marquis de Laplace/ Peter Simon Laplace

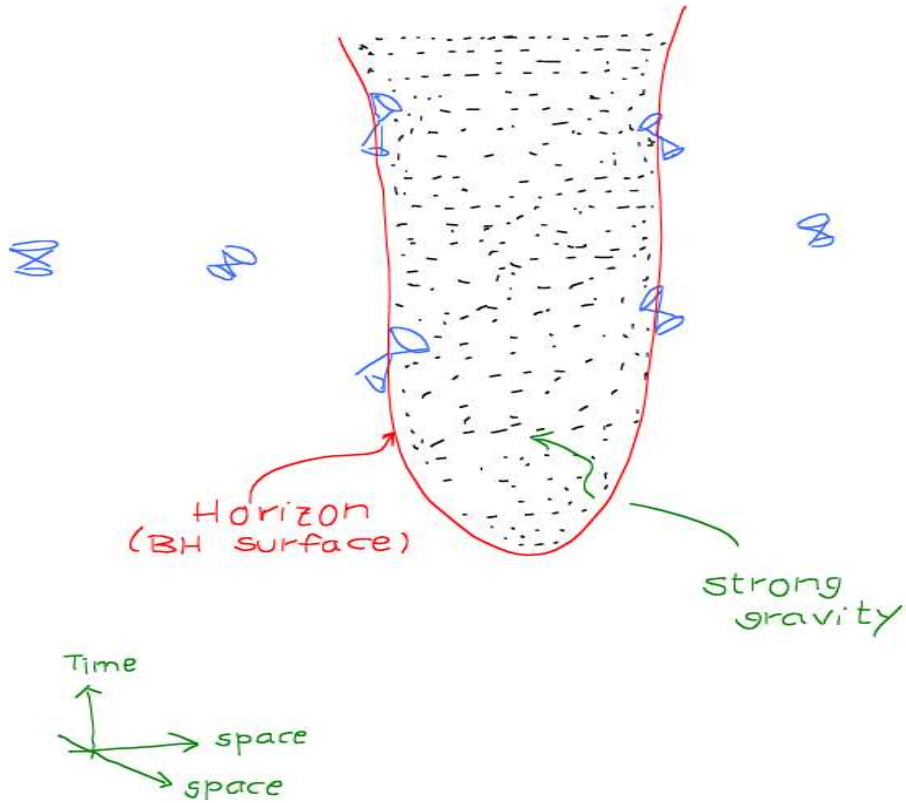
Exposition du système du Monde, Part II (1798/1799)



GENERAL RELATIVITY IS ESSENTIAL!

• Attractive as they seem, the Mitchell/Laplace arguments are conceptually flawed because speed of light is observer dependent in Newtonian physics. Strictly, No black holes in newtonian gravity.

• The notion of black holes requires an observer independent speed of light and gravity $\Rightarrow$ General Relativity is essential.



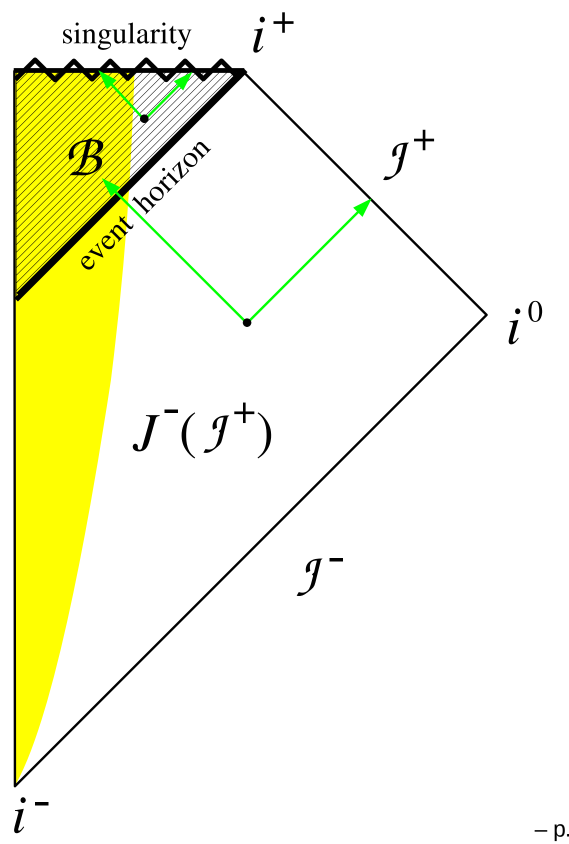
2. Horizons

- What exactly is a BH in GR? Precise definition?
  Event horizon! (Hawking, early 1970s)

- Idea: Black hole $\mathcal{B}$ is a region of space-time from which light cannot escape to infinity. More precisely: Penrose diagram.

$\mathcal{I}^+$ denotes future (null) infinity.

$J^-(\mathcal{I}^+)$ is the 'exterior space-time region' from which light can escape to $\mathcal{I}^+$. $\mathcal{B}$ is the black-hole region from where it cannot. Event horizon E is the outer boundary of $\mathcal{B}$. Once you cross E, cannot escape out! According to GR: Will crush into the singularity.



- In the Schwarzschild space-time, E is the $r = 2M$ surface. Curvature $\sim M/r^3 \sim 1/M^2$. Can be quite weak at E of super-massive black holes (weaker than that on the surface of earth!)

Unforeseen properties of event horizons E

General Relativity & Thermodynamics are related! Black holes of GR are subject to three laws: (Bardeen, Carter, Hawking)

i) Surface gravity κ is constant on E, if the BH is in equilibrium (stationary), even when E is non-spherical! ($\kappa \sim g$ on earth's surface)

ii) If a BH makes a transition from an equilibrium state to a nearby equilibrium state, the mass M of the BH, the area a of E, and κ, are related by

$$\delta M = \frac{\kappa}{8\pi G}\,\delta a + \delta[\text{Work done on the BH}]$$

iii) If matter satisfies 'energy conditions', the area a of E cannot decrease.

• Striking similarity with the laws of thermodynamics: (a multiple of) κ plays the role of temperature, and (a multiple of) a of entropy! (Bekenstein)

Event Horizons E and Thermodynamics

• However the analogy remained formal. Simple dimensional considerations $\Rightarrow$ cannot construct temperature from κ nor entropy from a in classical GR, i.e. with only G and c at one's disposal.

(Throughout, Boltzmann constant K set to 1.)

• Dramatic Change: Hawking's discovery that BHs radiate quantum mechanically as though they are black bodies at temperature $T = \hbar\kappa/2\pi$. From first law, one is led to assign entropy $S = a/4G\hbar = a/4\ell_{\rm Pl}^2$ to E.

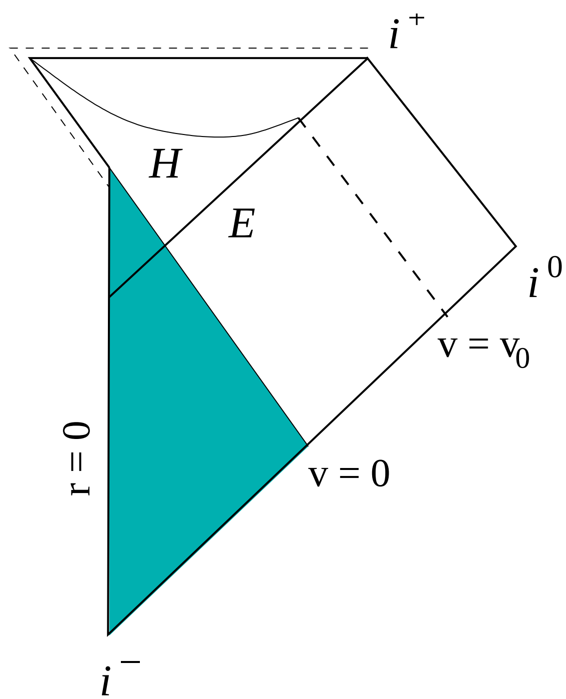
• The three pillars of fundamental physics, Quantum Mechanics, General Relativity and Statistical mechanics, unexpectedly brought together!

Dynamical BHs: ‘Spookiness’ of event horizons

• Event horizons are too global. Refer to the entire past of $\mathcal{I}^+$. A smooth change in space-time geometry in a small neighborhood of singularity can shift them drastically and even make them disappear! (Hajicek).

• Event horizons are teleological: One may be developing right now, in this room in anticipation of a gravitational collapse in our region of the Milky Way a million years from now!

Explicit solution to Einstein’s equation (Vaidya metric) in which E develops in a flat space-time region and grows in anticipation of a future gravitational collapse!



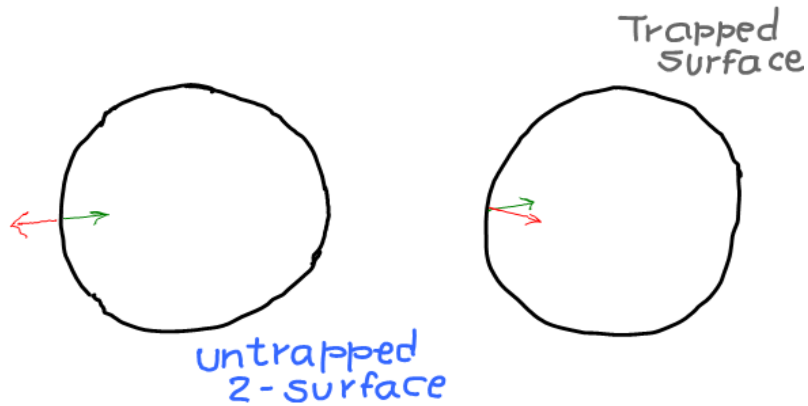
• Can not hope to generalize the first law to fully dynamical situations using E: Area can grow even when nothing is falling across E!

• To construct E, one needs to know full space-time. Not very useful to characterize a BH during numerical simulations.

Quasi-local horizons

• To overcome limitations of E, quasi-local horizons were introduced more recently (Hayward, AA, Krishnan, Andersson, Galloway, Mars, Simon, ...).

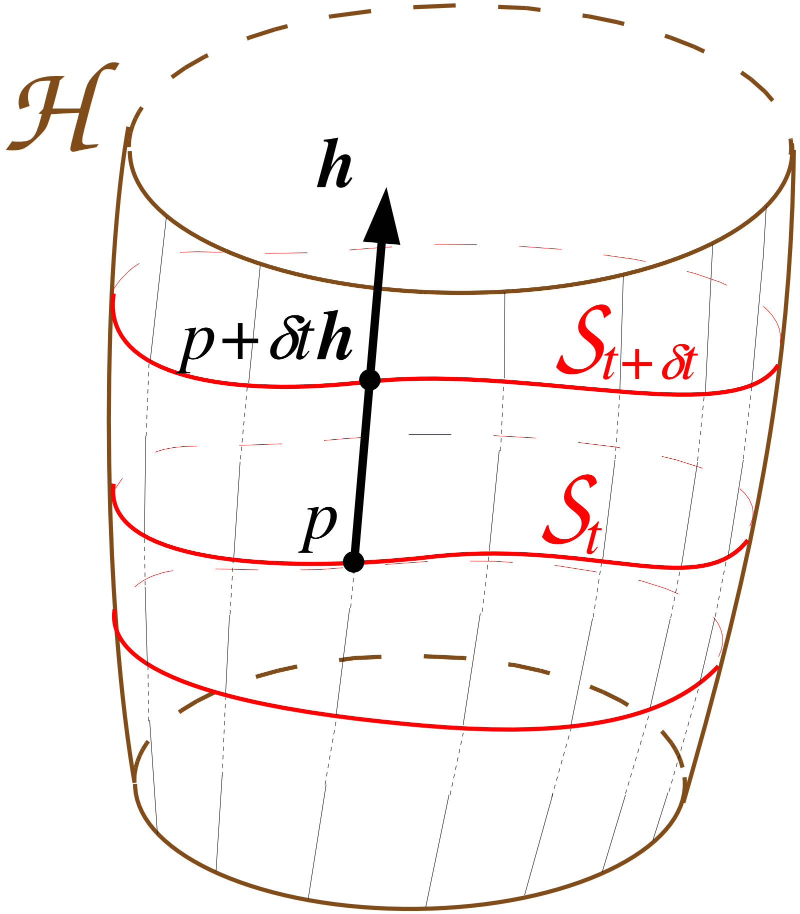
• Idea: Trapped surfaces. Boundary: marginally trapped surface, MTS S
Event horizons replaced by world tubes
$\mathcal{H}$ of MTSs obtained by stacking MTSs.



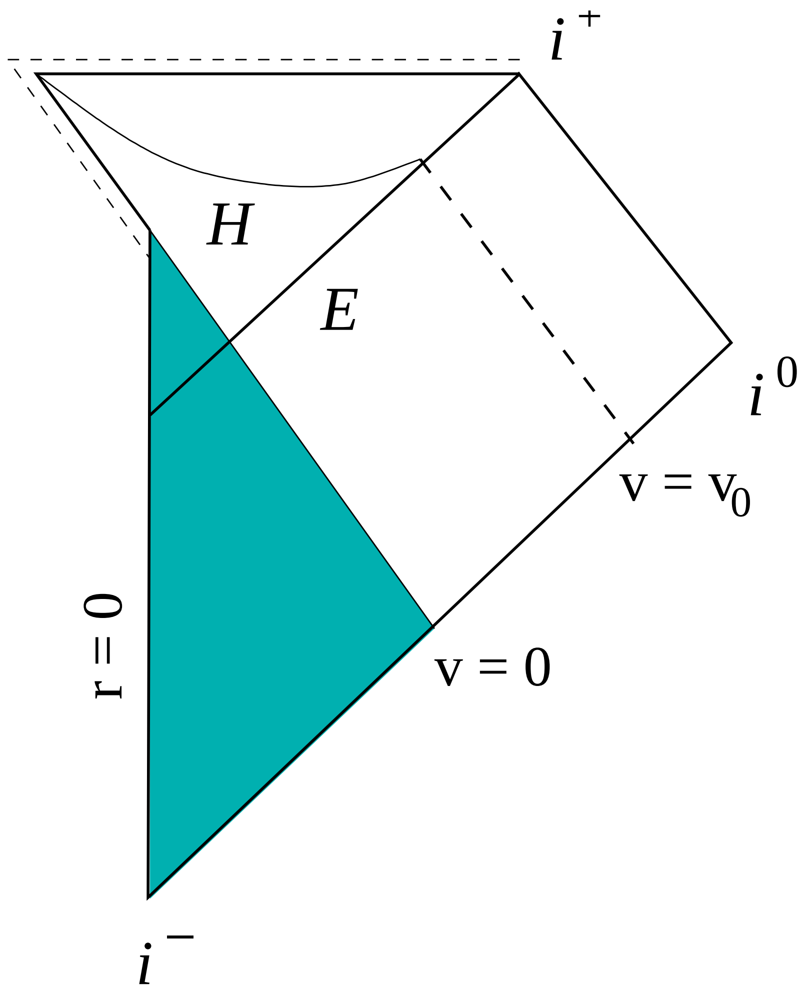
• Interesting cases for classical GR:
⋆ If $\mathcal{H}$ Space-like, area increases:
Dynamical horizon
Black hole is growing
by swallowing up matter & grav. Waves.

⋆ If $\mathcal{H}$ null, area constant:
Isolated horizon
Black hole has reached equilibrium.

Quasi-local horizons



(a) $\mathcal{H}$ obtained by stacking MTSs S_t



(b) Vaidya space-time

Isolated horizons and thermodynamics

• Quasi-local notions $\Rightarrow$ nothing teleological.
No quasi-local horizon in this room!

But what about Thermodynamics?? Extends to quasi-local horizons!
(AA, Krishnan, Beetle, Booth, Fairhurst, Hayward, Liko, Lewandowski,....)

• For isolated horizons, area constant: BH has reached equilibrium, although there may be dynamical processes far away. But zeroth law holds: surface gravity κ is constant.

• If we consider transition from one such equilibrium state to a nearby one, the first law again holds: $\delta M = (\kappa/8\pi G)\,\delta a + \delta\mathrm{work}$.

These laws hold even for hairy black holes, black holes with matter rings which distort the isolated horizon, ... New relations between solitons and colored black holes in Einstein-Yang-Mills-Higgs. theory; ...

Dynamical horizons and thermodynamics

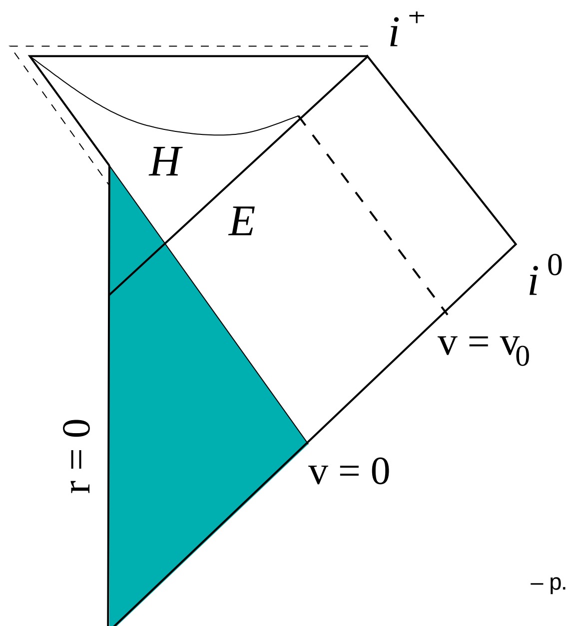
• In fully time-dependent processes: Dynamical horizons. Area increases. Entropy analogy restored.

Furthermore, area increase directly related to the influx of energy across $\mathcal{H}$! New integral law:

$$R(t_2) - R(t_1) = 2G \text{ [Flux of Energy]}$$

Energy Flux from both, matter and gravitational waves.
(R: Areal radius; $R = \sqrt{a/4\pi}$)

• Quasi-local horizons have many of the desirable properties of event horizons but are free from their drawbacks. Used routinely in computational relativity, mathematical physics and quantum gravity.



3. New Challenge: Black Hole Entropy

• First law of BH Mechanics + Hawking's discovery that $T_{\rm BH} = \kappa\hbar/2\pi \;\; \Rightarrow$ for isolated horizons, $S_{\rm BH} = a_{\rm hor}/4\ell_{\rm Pl}^2$

• Entropy: Why is the entropy proportional to area? For a $M_\odot$ black hole, we must have $\exp\, 10^{76}$ micro-states, a HUGE number even by standards of statistical mechanics. Where do these micro-states come from?

For gas in box, the microstates come from molecules; for a ferromagnet, from Heisenberg spins; Black hole ?
Cannot be gravitons: gravitational fields stationary.

• To answer these questions, must go beyond the classical space-time approximation used in the Hawking effect. Must take into account the quantum nature of gravity.

• Distinct approaches. Attractive features but none completely satisfactory. In Loop Quantum Gravity, this entropy arises from the huge number of microstates of the quantum horizon geometry. 'Atoms' of geometry itself!

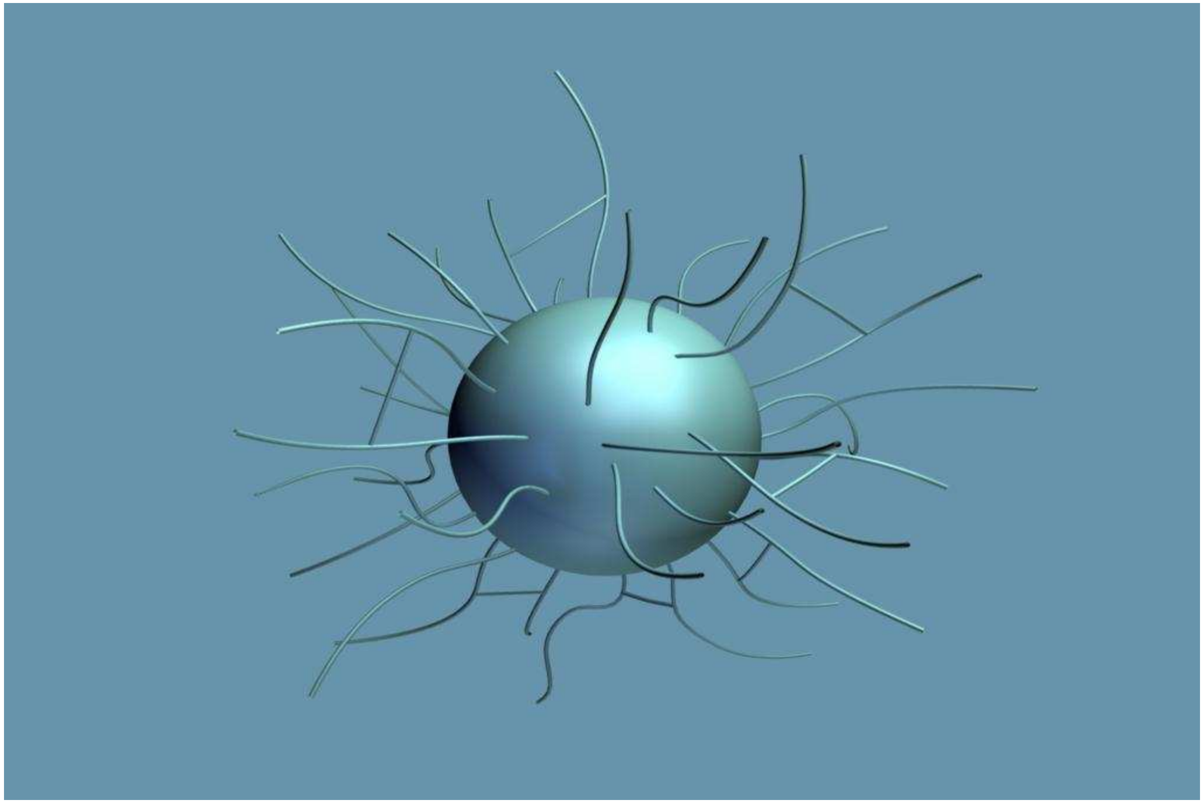
Quantum Horizon Geometry & Entropy

• Heuristics: Wheeler's It from Bit
Divide the horizon into elementary cells, each carrying area $\ell_{\rm Pl}^2$.
Assign to each cell a 'Bit' i.e. 2 states.
Then, # of cells $n \sim a_o/\ell_{\rm Pl}^2$; No of states $\mathcal{N} \sim 2^n$;
$S_{\rm hor} \sim \ln \mathcal{N} \sim n \ln 2 \sim a_o/\ell_{\rm Pl}^2$. Thus, $S_{\rm hor} \propto a_o/\ell_{\rm Pl}^2$.

• Argument made rigorous in quantum geometry. Many inaccuracies of the heuristic argument have to be overcome: Calculation has to know that the surface is black hole horizon; What is a quantum horizon? Isolated horizon boundary condition made into an operator equation. Quanta of area not $\ell_{\rm Pl}^2$ but $4\pi\gamma \sqrt{j(j+)}\, \ell_{\rm Pl}^2$.

• Interesting mathematical structures $\mathrm{U}(1)$ Chern-Simons theory; non-commutative torus, quantum $\mathrm{U}(1)$, mapping class group, ...(AA, Baez, Corichi, Krasnov; Domagala, Lewandowski; Meissner; AA, Engle, Van Den Broeck, ...)

Quantum Horizon



Continuum only an approximation. At Planck scale, fundamental excitations of bulk geometry 1-dimensional, polymer-like. Each quantum thread pierces the horizon $\mathcal{H}$ and deposits a quantum of area on $\mathcal{H}$. Quantum geometry of $\mathcal{H}$ described by the $\mathrm{U}(1)$ Chern-Simons theory.

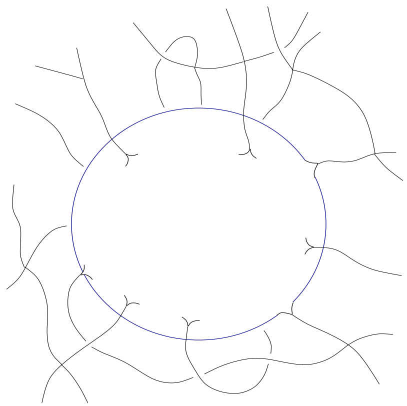
Quantum Horizon Geometry and Entropy

- Horizon geometry flat everywhere except at punctures. At punctures the bulk polymer excitations cause a 'tug' giving rise to quantized deficit angles. They add up to 4π providing a 2-sphere quantum geometry. 'Quantum Gauss-Bonnet Theorem'.

- As in Statistical mechanics, have to construct

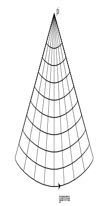
  a suitable ensemble ρ

  by specifying macroscopic parameters (multipoles) characterizing the horizon geometry. $S_{\rm hor} = {\rm Tr} \rho \ln \rho$ gives the log of the number of quantum horizon geometry states compatible with the classical geometry.



- $S_{\rm hor} = a_{\rm hor}/4\ell_{\rm Pl}^2 - (1/2) \ln(a_{\rm hor}/\ell_{\rm Pl}^2) + o(a_{\rm hor}/\ell_{\rm Pl}^2)$

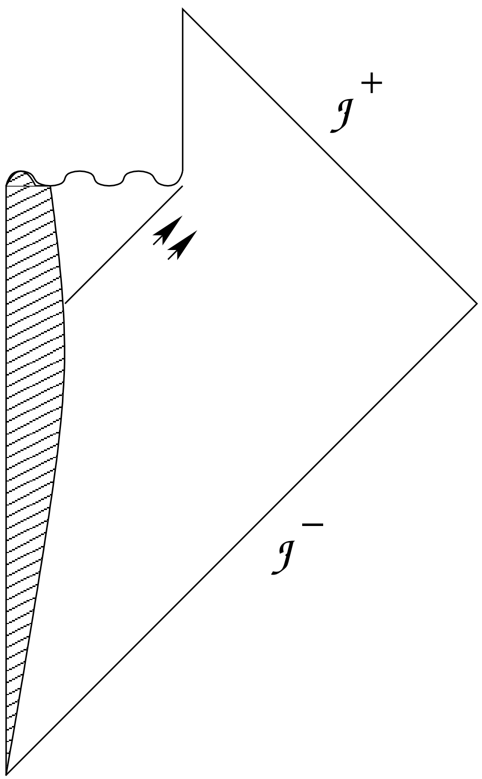
  for a specific value of the parameter γ.
  Procedure incorporates all physically interesting BHs and Cosmological horizons in one swoop.



4. Another Challenge: Information loss!

• Hawking radiation: Quantum Field Theory in a fixed BH space-time. Energy conservation $\Rightarrow$ BH must loose mass and evaporate.

• Suppose BH was formed by a collapsing matter in a pure state. Hawking radiation is thermal. So when the BH is 'gone', we are left with a mixed state of maximum entropy (for the given energy). So, in BH formation and evaporation, pure states seem to evolve to mixed states. Process seems non-unitary; information is lost! Suggests: Basic structure of quantum mechanics has to be modified!!

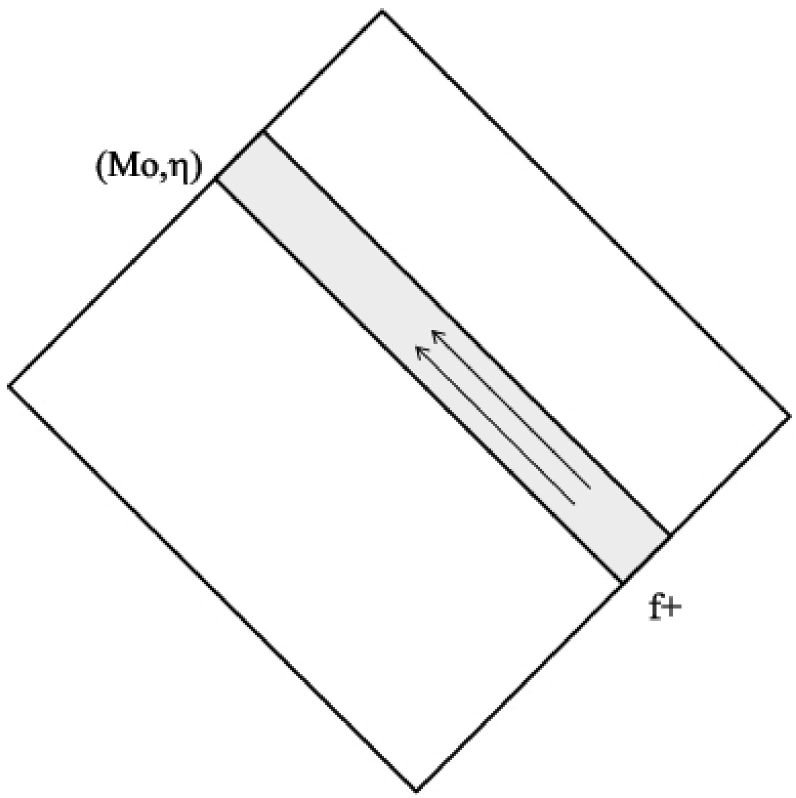


• Where did the information go?? Although BH has evaporated, in Hawking's picture a singularity still remains. (The Cheshire cat disappears but the smile remains!) Acts as a sink of information.

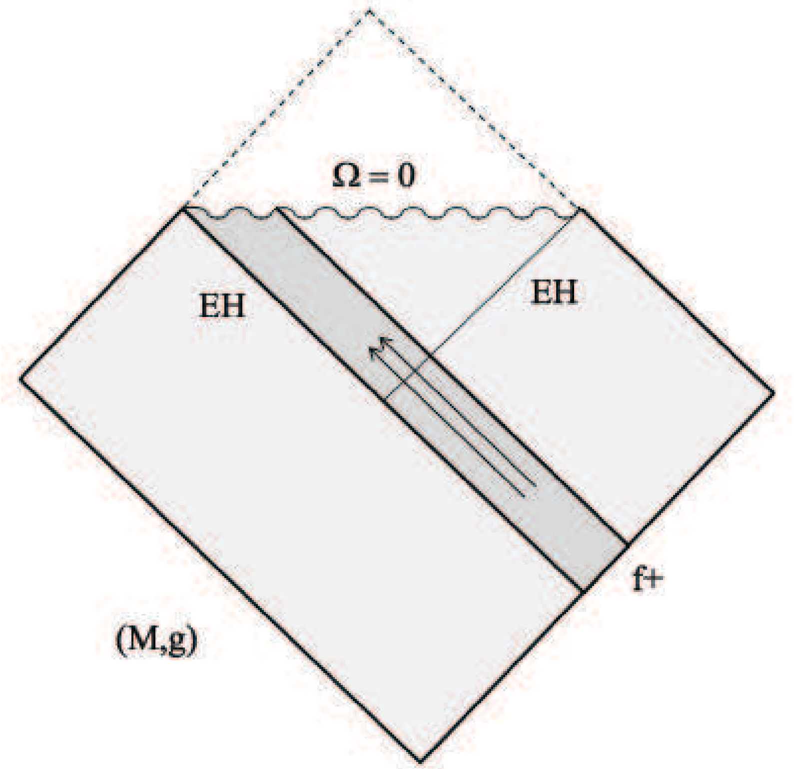
2-dimensional dilatonic BHs

• Problem still open! String theory suggests that pure states must evolve into pure states (AdS/CFT); information cannot be lost. But the reasoning requires $\Lambda < 0$ and as yet we do not know 'how the information comes out' in the space-time langauge.

• Recent progress: 2-dimensional CGHS BHs. (AA, Taveras, Varadarajan).

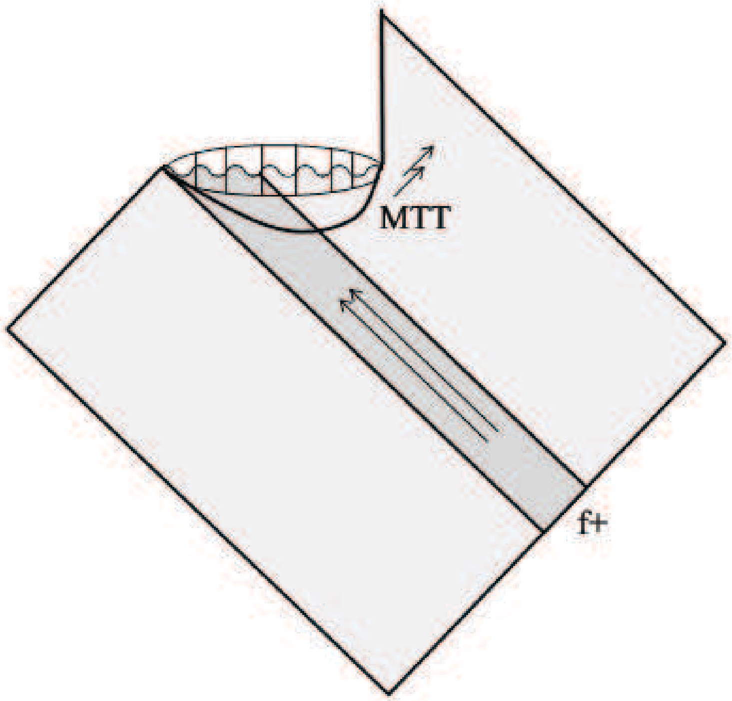


(c) Free scalar field in Mink space

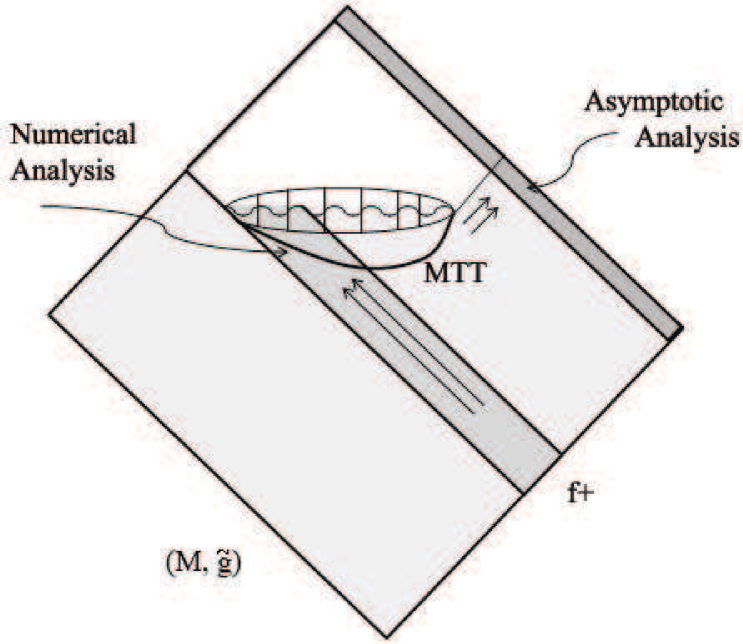


(d) 'collapse' leading to a BH

Quantum process: Older and the New descriptions



(e) Remnant singularity: sink of info



(f) No singularity & no info loss

There is no information loss because the quantum space-time is sufficiently larger than the classical one.

5. Summary

• A precise notion of black holes through event horizons was introduced over 35 years ago. Led to an astonishing connection between GR and Thermodynamics. To make it precise, need quantum physics! Beautiful convergence of ideas from the three pillars of fundamental physics.

• But event horizons are way too global and teleological. Euphoria followed by frustration.

• Quasi-local horizons. Free of these difficulties and provide even stronger analogs of the laws of thermodynamics. have had applications to computational relativity, mathematical physics and quantum gravity.

• Again, new vexing questions: Why is the entropy proportional to the Isolated horizon area? Is information lost during BH formation and evaporation?

New Challenges!

• Several ways of accounting for BH entropy have emerged. (String theory, Loop quantum gravity). Loop quantum gravity: BH entropy provides a theoretical microscope to zoom in on properties of quantum geometry on the Planck scale. Several fresh and fascinating developments. However, issue is not fully settled: None of the approaches is completely satisfactory. Moreover, underlying ideas very different. Are these special cases of a more general derivation?

• Information loss issue: Consensus has turned $180^{\rm deg}$ around. Now, a majority view is that information is not lost. Insights into how it is recovered provided by recent analysis of 2-d black holes. But complete picture still beyond our grasp.

• Black holes seem to have a vast potential to tease us, vex us and then lead us to deep new insights. But euphoria is quickly followed by new vexing puzzles and challenges!!

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