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STRONG CONVERGENCE THEOREMS FOR ITERATIVE SCHEMES WITH ERRORS FOR ASYMPTOTICALLY DEMICONTRACTIVE MAPPINGS IN ARBITRARY REAL NORMED LINEAR SPACES

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ABSTRACT. In the present paper, by virtue of new analysis technique, we will establish several strong convergence theorems for the modified Ishikawa and Mann iteration schemes with errors for a class of asymptotically demicontractive mappings in arbitrary real normed linear spaces. Our results extend, generalize and improve the corresponding results obtained by Igbokwe [1], Liu [2], Osilike [3] and others.

Key words and phrases: Asymptotically demicontractive mapping; Modified Mann and Ishikawa iteration schemes with errors; arbitrary linear space.

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1. INTRODUCTION

Let X be a real normed linear space and let J denote the *normalized duality mapping* from X into 2^{X^*} given by

$$J(x) = \{f \in X^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\}, \quad x \in X,$$

where X^* denotes the dual space of X and $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing of elements between X and X^*.

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Let $F(T)$ denote the set of all fixed points of a mapping T. Let C be a nonempty subset of X.

A mapping $T : C \to C$ is said to be *k-strictly asymptotically pseudocontractive* with a sequence $\{k_n\} \subset [0,\infty)$, $k_n \geq 1$ and $k_n \to 1$ as $n \to \infty$ if there exists $k \in [0,1)$ such that

$$\|T^n x - T^n y\|^2 \leq k_n^2 \|x-y\|^2 + k\, \|(x - T^n x) - (y - T^n y)\|^2 \tag{1.1}$$

for all $n \geq 1$ and $x, y \in C$.

The mapping T is said to be *asymptotically demicontractive* with a sequence $\{k_n\} \subset [1,\infty)$ and $\lim_{n\to\infty} k_n = 1$ if $F(T) \neq \emptyset$ and there exists $k \in [0,1)$ such that

$$\|T^n x - p\|^2 \leq k_n^2 \|x-p\|^2 + k\|x - T^n x\|^2 \tag{1.2}$$

for all $n \geq 1$, $x \in C$ and $p \in F(T)$.

The classes of k-strictly asymptotically pseudocontractive and asymptotically demicontractive mappings, as a natural extension to the class of asymptotically non-expansive mappings, were first introduced in Hilbert spaces by Liu [5] in 1996. By using the modified Mann iterates introduced by Schu [4, 5], he established several strong convergence results concerning an iterative approximation to fixed points of k-strictly asymptotically pseudocontractive and asymptotically demicontractive mappings in Hilbert spaces. In 1998, Osilike [3], by virtue of normalized duality mapping, first extended the concepts of k-strictly asymptotically pseudocontractive and asymptotically demicontractive maps from Hilbert spaces to the much more general Banach spaces, and then proved the corresponding convergence theorems which generalized the results of Liu [2].

A mapping $T : C \to C$ is said to be *k-strictly asymptotically pseudocontractive* with a sequence $\{k_n\} \subset [0,\infty)$, $k_n \geq 1$ and $k_n \to 1$ as $n \to \infty$ if there exist $k \in [0,1)$ and $j(x-y) \in J(x-y)$ such that

$$\begin{aligned} &\langle (I - T^n)x - (I - T^n)y, j(x-y)\rangle \\ &\qquad \geq \frac{1}{2}(1-k)\|(I-T^n)x - (I-T^n)y\|^2 - \frac{1}{2}(k_n^2 - 1)\|x-y\|^2 \end{aligned} \tag{1.3}$$

for all $n \geq 1$ and $x, y \in C$.

The mapping T is called an *asymptotically demicontractive mapping* with a sequence $\{k_n\} \subset [0,\infty)$, $\lim_{n\to\infty} k_n = 1$ if $F(T) \neq \emptyset$ and there exist $k \in [0,1)$ and $j(x-y) \in J(x-y)$ such that

$$\langle x - T^n x, j(x-p)\rangle \geq \frac{1}{2}(1-k)\|x - T^n x\|^2 - \frac{1}{2}(k_n^2-1)\|x-p\|^2 \tag{1.4}$$

for all $n \geq 1$, $x \in C$ and $p \in F(T)$.

Furthermore, T is said to be *uniformly L-Lipschitzian* if there is a constant $L \geq 1$ such that

$$\|T^n x - T^n y\| \leq L\|x-y\| \tag{1.5}$$

for all $x, y \in C$ and $n \geq 1$.

Remark 1.1. The definitions above may be stated in the setting of a real normed linear space. In the case of X being a Hilbert space, (1.1) and (1.2) are equivalent to (1.3) and (1.4), respectively.

Recall that there are two iterative schemes with errors which have been used extensively by various authors.

Let X be a normed linear space, C be a nonempty convex subset of X and $T : C \to C$ be a given mapping. Then the *modified Ishikawa iteration scheme* $\{x_n\}$ *with errors* is defined by

$$\begin{cases} x_1 \in C, \\ y_n = (1-\beta_n)x_n + \beta_n T^n(x_n) + v_n, & n \geq 1, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T^n(y_n) + u_n, & n \geq 1, \end{cases}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ are some suitable sequences in $[0,1]$ and $\{u_n\}$, $\{v_n\}$ are two summable sequences in X.

With X, C, $\{\alpha_n\}$ and x_1 as above, the *modified Mann iteration scheme* $\{x_n\}$ *with errors* is defined by

$$\begin{cases} x_1 \in C, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T^n(x_n) + u_n, & n \geq 1. \end{cases} \tag{1.6}$$

Let X, C and T be as in above. Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{a'_n\}$, $\{b'_n\}$ and $\{c'_n\}$ be real sequences in $[0,1]$ satisfying $a_n + b_n + c_n = 1 = a'_n + b'_n + c'_n$ and let $\{u_n\}$ and $\{v_n\}$ be bounded sequences in C. Define the *modified Ishikawa iteration schemes* $\{x_n\}$ *with errors* generated from an arbitrary $x_1 \in C$ as follows:

$$\begin{cases} y_n = a_n x_n + b_n T^n(x_n) + c_n u_n, & n \geq 1, \\ x_{n+1} = a'_n x_n + b'_n T^n y_n + c'_n v_n, & n \geq 1. \end{cases} \tag{1.7}$$

In particular, if we set $b_n = c_n = 0$ in (1.7), we obtain the modified Mann iteration scheme $\{x_n\}$ with errors given by

$$\begin{cases} x_1 \in C, \\ x_{n+1} = a'_n x_n + b'_n T^n x_n + c'_n v_n, & n \geq 1. \end{cases} \tag{1.8}$$

Osilike [3] proved the following convergence theorems for k-strictly asymptotically demicontractive mappings:

Theorem 1.2. [3] *Let* $q > 1$ *and let* E *be a real* q*-uniformly smooth Banach space. Let* K *be a nonempty closed convex and bounded subset of* E *and* $T : K \to K$ *be a completely continuous and uniformly* L*-Lipschitzian asymptotically demicontractive mapping with a sequence* $\{k_n\} \subset [1,\infty)$ *for all* $n \geq 1$, $k_n \to 1$ *as* $n \to \infty$ *and* $\sum_{n=1}^{\infty}({k_n}^2 - 1) < \infty$. *Let* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *be real sequences in* $[0,1]$ *satisfying the conditions:*

(i) $0 < \epsilon \leq c_q {\alpha_n}^{q-1} \leq \frac{1}{2}\{q(1-k)(1+L)^{-(q-2)}\} - \epsilon$ *for all* $n \geq 1$ *and for some* $\epsilon > 0$,
(ii) $\sum_{n=1}^{\infty} \beta_n < \infty$.

Then the sequence $\{x_n\}$ *defined by* (1.6) *with* $u_n \equiv 0$ *and* $v_n \equiv 0$ *for all* $n \geq 1$ *converges strongly to a fixed point of* T.

Very recently, Igbokwe [2] extended the above Theorem 1.2 to Banach spaces. More precisely, he proved the following results:

Theorem 1.3. [2] *Let* E *be a real Banach space and* K *be a nonempty closed convex subset of* E. *Let* $T : K \to K$ *be a completely continuous and uniformly* L*-Lipschitzian asymptotically demicontractive mapping with a sequence* $\{k_n\} \subset [1,\infty)$ *for all* $n \geq 1$, $k_n \to 1$ *as* $n \to \infty$ *and* $\sum_{n=1}^{\infty}(k_n^2 - 1) < \infty$. *Let the sequence* $\{x_n\}$ *be defined by* (1.7) *with the restrictions that*

(i) $a_n + b_n + c_n = 1 = a'_n + b'_n + c'_n$,

(ii) $\sum_{n=0}^{\infty} b'_n = \infty$,
(iii) $\sum_{n=0}^{\infty} (b')^2 < \infty$, $\sum_{n=0}^{\infty} c'_n < \infty$, $\sum_{n=0}^{\infty} b_n < \infty$ *and* $\sum_{n=0}^{\infty} c_n < \infty$.

Then the modified Ishikawa iteration $\{x_n\}$ *defined by* (1.6) *and* (1.7) *converges strongly to a fixed point* p *of* T.

It is our purpose in this paper to extend and improve the above Theorem 1.3 from Banach spaces to real normed linear spaces. In the case of Banach spaces, we use the Condition (A) to replace the assumption that T is completely continuous.

In the sequel, we will need the following lemmas:

Lemma 1.4. *Let* X *be a normed linear space and* C *be a nonempty convex subset of* X. *Let* $T : C \to C$ *be a uniformly* L*-Lipschitzian mapping and the sequence* $\{x_n\}$ *be defined by* (1.7). *Then we have*

$$\begin{aligned} \|Tx_n - x_n\| \le \|T^n x_n - x_n\| &+ L(1+L^2)\|T^{n-1}x_{n-1} - x_{n-1}\| \\ &+ L(1+L)c'_{n-1}\|v_{n-1} - x_{n-1}\| \\ &+ L^2(1+L)c_{n-1}\|u_{n-1} - x_n\| \\ &+ Lc'_{n-1}\|x_{n-1} - T^{n-1}x_{n-1}\|, \quad n \ge 1. \end{aligned} \tag{1.9}$$

Proof. See Igbokwe [1, Lemma 1]. □

Lemma 1.5. *Let* X *be a normed linear space and* C *be a nonempty convex subset of* X. *Let* $T : C \to C$ *be a uniformly* L*-Lipschitzian and asymptotically demicontractive mapping with a sequence* $\{k_n\}$ *such that* $k_n \ge 1$ *and* $\sum_{n=1}^{\infty}(k_n - 1) < \infty$. *Let the sequence* $\{x_n\}$ *be defined by* (1.7) *with the restrictions*

$$\sum_{n=1}^{\infty} b'_n = \infty, \quad \sum_{n=1}^{\infty} (b'_n)^2 < \infty, \quad \sum_{n=1}^{\infty} c'_n < \infty, \quad \sum_{n=1}^{\infty} c_n < \infty.$$

Then we have the following conclusions:

(i) $\lim_{n\to\infty} \|x_n - p\|$ *exists for any* $p \in F(T)$.
(ii) $\lim_{n\to\infty} d(x_n, F(T))$ *exists.*
(iii) $\liminf_{n\to\infty} \|x_n - Tx_n\| = 0$.

Proof. It is very clear that (1.7) is equivalent to the following:

$$\begin{cases} y_n = (1-b_n)x_n + b_n T^n(x_n) + c_n(u_n - x_n), & n \ge 1, \\ x_{n+1} = (1-b'_n)x_n + b'_n T^n y_n + c'_n(v_n - x_n), & n \ge 1. \end{cases} \tag{1.10}$$

For any $p \in F(T)$, let $M > 0$ be such that

$$M = \max\{\sup\{\|u_n - p\|\}, \sup\{\|v_n - p\|\}\}.$$

Observe first that

$$\begin{aligned} \|y_n - p\| &\le (1-b_n)\|x_n - p\| + b_n L\|x_n - p\| + c_n(M + \|x_n - p\|) \\ &\le (1+L)\|x_n - p\| + M \end{aligned} \tag{1.11}$$

and

$$\begin{aligned} \|T^n y_n - x_n\| &\le L\|y_n - p\| + \|x_n - p\| \\ &\le L[(1+L)\|x_n - p\| + M] + \|x_n - p\| \\ &\le [1 + L(1+L)]\|x_n - p\| + ML. \end{aligned} \tag{1.12}$$

Observe also that

$$\begin{aligned}(1.13)\qquad \|x_{n+1}-y_n\| &\le \|x_n-y_n\| + b'_n[\|T^ny_n-p\|+\|y_n-p\|] \\ &\qquad + c'_n[\|v_n-p\|+\|y_n-p\|] \\ &\le [b_n(1+L)+c_n]\|x_n-p\| + (c_n+c'_n)M \\ &\qquad + [b'_n(1+L)+c'_n][(1+L)\|x_n-p\|+M] \\ &\le \sigma_n\|x_n-p\|+\varsigma_n,\end{aligned}$$

where

$$\sigma_n = [b_n(1+L)+c_n] + [b'_n(1+L)+c'_n](1+L)$$

and

$$\varsigma_n = M[b'_n(1+L)+2c'_n+c_n].$$

Thus we have

$$\begin{aligned}(1.14)\qquad \|x_{n+1}-x_n\| &\le b'_n\|T^ny_n-x_n\| + c'_n(M+\|x_n-p\|) \\ &\le b'_n\sigma_n\|x_n-p\|+\varsigma_n+c'_n(M+\|x_n-p\|).\end{aligned}$$

Using iterates (1.10), we have

$$\begin{aligned}(1.15)\qquad \|x_{n+1}-p\|^2 &\le \|x_n-p\|\|x_{n+1}-p\| - b'_n\langle x_n-T^ny_n, j(x_{n+1}-p)\rangle \\ &\qquad + c'_n\langle v_n-x_n, j(x_{n+1}-p)\rangle\| \\ &\le \frac{1}{2}\|x_n-p\|^2+\frac{1}{2}\|x_{n+1}-p\|^2 - b'_n\langle x_{n+1}-T^nx_{n+1}, j(x_{n+1}-p)\rangle \\ &\qquad + b'_n\langle x_{n+1}-x_n+T^ny_n-T^nx_{n+1}, j(x_{n+1}-p)\rangle \\ &\qquad + c'_n(M+\|x_n-p\|)\|x_{n+1}-p\|,\end{aligned}$$

which implies that

$$\begin{aligned}(1.16)\qquad \|x_{n+1}-p\|^2 &\le \|x_n-p\|^2-(1-k)b'_n\|x_{n+1}-T^nx_{n+1}\|^2 \\ &\qquad + b'_n(k_n^2-1)\|x_{n+1}-p\|^2+2b'_n(\|x_{n+1}-x_n\| \\ &\qquad + \|T^nx_{n+1}-T^ny_n\|)\|x_{n+1}-p\| \\ &\qquad + 2c'_n(M+\|x_n-p\|)\|x_{n+1}-p\|.\end{aligned}$$

Substituting (1.12) – (1.14) in (1.16) and, after some calculations, we obtain

$$(1.17)\qquad \|x_{n+1}-p\|^2 \le (1+\gamma_n)\|x_n-p\|^2-(1-k)b'_n\|x_{n+1}-T^nx_{n+1}\|^2$$

for all very large n, where the sequence $\{\gamma_n\}$ satisfies that $\sum_{n=1}^{\infty}\gamma_n<\infty$. A direct induction of (1.17) leads to

$$(1.18)\qquad \|x_{n+1}-p\|^2 \le \|x_n-p\|^2+M\gamma_n-b'_n\|x_{n+1}-T^nx_{n+1}\|^2,$$

which implies that $\lim_{n\to\infty}\|x_n-p\|$ exists by Tan and Xu [7, Lemma 1] and so this proves the claim (i). The claim (ii) follows from (1.18).

Now, we prove the claim (iii). It follows from (1.18) that

$$\sum_{n=1}^{\infty} b'_n\|x_{n+1}-T^nx_{n+1}\|^2<\infty$$

and hence

$$\liminf_{n\to\infty}\|x_{n+1}-T^nx_{n+1}\|=0$$

since $\sum_n b'_n=\infty$. Therefore, we have $\liminf_{n\to\infty}\|x_n-T^nx_n\|=0$ by (1.7) and so $\liminf_{n\to\infty}\|x_n-Tx_n\|=0$ by Lemma 1.4. This completes the proof. □

A mapping $T : C \to C$ with a nonempty fixed point set $F(T)$ in C will be said to satisfy the *Condition* (A) on C if there is a nondecreasing function $f : [0,\infty) \to [0,\infty)$ with $f(0) = 0$ and $f(r) > 0$ for all $r \in (0,\infty)$ such that

$$\|x - Tx\| \geq f(d(x, F(T)))$$

for all $x \in C$.

2. THE MAIN RESULTS

Now we prove the main results of this paper.

Theorem 2.1. *Let X be a real normed linear space, C be a nonempty closed convex subset of X and $T : C \to C$ be a completely continuous and uniformly L-Lipschitzian asymptotically demicontractive mapping with a sequence $\{k_n\} \subset [1,\infty)$ such that $k_n \geq 1$ and $\sum_{n=1}^{\infty}(k_n - 1) < \infty$. Let the sequence $\{x_n\}$ be defined by (1.7) with the restrictions*

$$\sum_{n=1}^{\infty} b'_n = \infty, \quad \sum_{n=1}^{\infty} {b'_n}^2 < \infty, \quad \sum_{n=1}^{\infty} c'_n < \infty, \quad \sum_{n=1}^{\infty} c_n < \infty.$$

Then $\{x_n\}$ converges strongly to a fixed point p of T.

Proof. It follows from Lemma 1.5 that

$$\liminf_{n\to\infty} \|x_n - Tx_n\| = 0.$$

Since T is completely continuous, we see that there exists an infinite subsequence $\{x_{n_k}\}$ such that $\{x_{n_k}\}$ converges strongly for some $p \in C$ and $Tp = p$. This shows that $p \in F(T)$. However, $\lim_{n\to\infty} \|x_n - p\|$ exists for any $p \in F(T)$ and so we must have that the sequence $\{x_n\}$ converges strongly to p. This completes the proof. □

Theorem 2.2. *Let X be a real Banach space, C be a nonempty closed convex subset of X and $T : C \to C$ be a uniformly L-Lipschitzian and asymptotically demicontractive mapping with a sequence $\{k_n\} \subset [1,\infty)$ such that $k_n \geq 1$ and $\sum_{n=1}^{\infty}(k_n - 1) < \infty$. Let the sequence $\{x_n\}$ be defined by (1.7) with the restrictions*

$$\sum_{n=1}^{\infty} b'_n = \infty, \quad \sum_{n=1}^{\infty} {b'_n}^2 < \infty, \quad \sum_{n=1}^{\infty} c'_n < \infty, \quad \sum_{n=1}^{\infty} c_n < \infty.$$

Suppose in addition that T satisfies the Condition (A), then the sequence $\{x_n\}$ converges strongly to a fixed point p of T.

Proof. By Lemma 1.5, we see that

$$\liminf_{n\to\infty} \|x_n - Tx_n\| = 0.$$

Since T satisfies the Condition (A), we have

$$\liminf_{n\to\infty} f(d(x_n, F(T))) = 0$$

and hence

$$\liminf_{n\to\infty} d(x_n, F(T)) = 0.$$

By Lemma 1.5 (ii), we conclude that $d(x_n, F(T)) \to 0$ as $n \to \infty$.

Now we can take an infinite subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and a sequence $\{p_j\} \subset F(T)$ such that $\|x_{n_j} - p_j\| \leq 2^{-j}$. Set $M = \exp\{\sum_{n=1}^{\infty} \gamma_n\}$ and write $n_{j+1} = n_j + l$ for some $l \geq 1$. Then we have

$$\begin{aligned} \|x_{n_{j+1}} - p_j\| &= \|x_{n_j+l} - p_j\| \\ &\leq [1 + \gamma_{n_j+l-1}]\|x_{n_j+l-1} - p_j\| \\ &\leq \exp\left\{\sum_{m=0}^{l-1} \gamma_{n_j+m}\right\} \|x_{n_j} - p_j\| \\ &\leq \frac{M}{2^j}. \end{aligned} \tag{2.1}$$

It follows from (2.1) that

$$\|p_{j+1} - p_j\| \leq \frac{2M+1}{2^{j+1}}.$$

Hence $\{p_j\}$ is a Cauchy sequence. Assume that $p_j \to p$ as $j \to \infty$. Then $p \in F(T)$ since $F(T)$ is closed and this in turn implies that $x_j \to p$ as $j \to \infty$. This completes the proof. □

Remark 2.3. We remark that, if $T : C \to C$ is completely continuous, then it must be demicompact (cf. [6]) and, if T is continuous and demicompact, it must satisfy the Condition (A) (cf. [6]). In view of this observation, our Theorem 2.1 improves Theorem 1.3 in the following aspects:

(i) X may be not a Banach space.
(ii) T may be not completely continuous.
(iii) Our proof methods are simpler than those of Igbokwe [1, Theorem 2].

As corollaries of Theorems 2.1 and 2.2, we have the following:

Corollary 2.4. *Let X be a real normed linear space, C be a nonempty closed convex subset of X and $T : C \to C$ be a completely continuous and uniformly L-Lipschitzian asymptotically demicontractive mapping with a sequence $\{k_n\} \subset [0, \infty)$ such that $k_n \geq 1$ and $\sum_{n=1}^{\infty}(k_n - 1) < \infty$. Let the sequence $\{x_n\}$ be defined by (1.8) with the restrictions*

$$\sum_{n=1}^{\infty} b_n' = \infty, \quad \sum_{n=1}^{\infty} {b_n'}^2 < \infty, \quad \sum_{n=1}^{\infty} c_n' < \infty.$$

Then $\{x_n\}$ converges strongly to a fixed point p of T.

Proof. By taking $b_n, c_n \equiv 0$ for $n \geq 1$ in Theorem 2.1, we can obtain the desired conclusion. □

Corollary 2.5. *Let X be a real Banch space, C be a nonempty closed convex subset of X and $T : C \to C$ be a uniformly L-Lipschitzian asymptotically demicontractive mapping with a sequence $\{k_n\} \subset [0, \infty)$ such that $k_n \geq 1$ and $\sum_{n=1}^{\infty}(k_n - 1) < \infty$. Let the sequence $\{x_n\}$ be defined by (1.8) with the restrictions*

$$\sum_{n=1}^{\infty} b_n' = \infty, \quad \sum_{n=1}^{\infty} b_n^2 < \infty, \quad \sum_{n=1}^{\infty} c_n' < \infty.$$

If T satisfies the Condition (A) on the sequence $\{x_n\}$, then $\{x_n\}$ converges strongly to a fixed point p of T.

Proof. It follows from Theorem 2.2 by taking $b_n, c_n \equiv 0$ for all $n \geq 1$. □

Remark 2.6. Using the same methods as in Lemma 1.5, Theorems 2.1 and 2.2, we can prove several convergence results similar to Theorems 2.1 and 2.2 concerning on the modified Ishikawa iteration schemes with errors defined by (1.6).

Remark 2.7. Igbokwe [1, Corollary 1] has shown that, if $T : C \to C$ is asymptotically pseudocontractive, then it must be uniformly L-Lipschitzian and hence our Theorems 2.1 and 2.2 hold for asymptotically pseudocontractive mappings with a nonempty fixed point set $F(T)$.

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