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REPORT OF THE GENERAL INEQUALITIES 8 CONFERENCE SEPTEMBER 15–21, 2002, NOSZVAJ, HUNGARY

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1. INTRODUCTION

The General Inequalities meetings have a long tradition extending to almost thirty years. The first 7 meetings were held in the Mathematical Research Institute at Oberwolfach. The 7th meeting was organized in 1995. Due to the long time having elapsed since this meeting and the growing interest in inequalities, the Scientific Committee of GI7 (consisting of Professors Catherine Bandle (Basel), W. Norrie Everitt (Birmingham), László Losonczi (Debrecen), and Wolfgang Walter (Karlsruhe)) agreed that the 8th General Inequalities meeting be held in Hungary. It took place from September 15 to 21, 2002, at the De La Motte Castle in Noszvaj and was organized by the Institute of Mathematics and Informatics of the University of Debrecen.

The Scientific Committee of GI8 consisted of Professors Catherine Bandle (Basel), László Losonczi (Debrecen), Michael Plum (Karlsruhe), and Wolfgang Walter (Karlsruhe) as Honorary Member.

The Local Organizing Committee consisted of Professors Zoltán Daróczy, Zsolt Páles, and Attila Gilányi as Secretary, The Committee Members were ably assisted by Mihály Bessenyei, Borbála Fazekas, and Attila Házy.

The 36 participants came from Australia (4), Canada (1), Czech Republic (1), Germany (4), Hungary (9), Japan (2), Poland (3), Romania (3), Switzerland (2), Sweden (3), United Kingdom (1), and the United States of America (3).

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Professor Walter opened the Symposium on behalf of the Scientific Committee. Professor Páles then welcomed the participants on behalf of the Local Organizing Committee.

The talks at the symposium focused on the following topics: convexity and its generalizations; mean values and functional inequalities; matrix and operator inequalities; inequalities for ordinary and partial differential operators; integral and differential inequalities; variational inequalities.

A number of sessions were, as usual, devoted to problems and remarks.

On the evening of Tuesday, September 17, the Gajdos Band performed Hungarian Folk Music which was received with great appreciation.

On Wednesday, the participants visited the Library and Observatory of the Eszterházy College of Eger and the famous fortress of the city. The excursion concluded with a dinner in Eger.

The scientific sessions were followed on Thursday evening by a festive banquet in the De La Motte Castle. The conference was closed on Friday by Professor Catherine Bandle.

Abstracts of the talks are in alphabetical order of the authors. These are followed by the problems and remarks (in approximate chronological order), two addenda to earlier GI volumes, and finally, the list of participants. In the cases where multiple authors are listed, the talk was presented by the first named author.

2. ABSTRACTS

TSUYOSHI ANDO

Löwner Theorem of Indefinite Type

ABSTRACT

The most familiar form of the Löwner theorem on matrices says that $A \geq B \geq 0$ implies $A^{\frac{1}{2}} \geq B^{\frac{1}{2}}$. Here $A \geq B$ means the Löwner ordering, that is, both A and B are Hermitian and $A - B$ is positive semidefinite.

We will show that if both A and B have only non-negative eigenvalues and J is an (indefinite) Hermitian involution then $JA \geq JB$ implies $JA^{\frac{1}{2}} \geq JB^{\frac{1}{2}}$.

We will derive this as a special case of the following result. If a real valued function $f(t)$ on $[0, \infty)$ is matrix-monotone of all order in the sense of Löwner then $JA \geq JB$ implies $J \cdot f(A) \geq J \cdot f(B)$. Here $f(A)$ is defined by usual functional calculus.

The classical Löwner theorem shows that $t^{\frac{1}{2}}$ is matrix-monotone of all order.

CATHERINE BANDLE

Rayleigh-Faber-Krahn Inequalities and Auasilinear Boundary Value Problems

ABSTRACT

The classical Rayleigh-Faber-Krahn inequality states that among all domains of given area the circle has the smallest principal frequency. The standard proof is by Schwarz symmetrization. This technique extends to higher dimensions and to the best Sobolev constants. For weighted Sobolev constants symmetrization doesn't apply. In this talk we propose a substitute. Emphasis is put on the case with the critical exponent. As an application we derive L_∞-bounds for Emden type equations involving the p-Laplacian.

SORINA BARZA

Duality Theorems Over Cones of Monotone Functions in Higher Dimensions

ABSTRACT

Let f be a non-negative function defined on $\mathbb{R}_+^n$ which is monotone in each variable separately. If $1 < p < \infty$, $g \geq 0$ and v a product weight, then equivalent expression for

$$\sup \frac{\int_{\mathbb{R}_+^n} fg}{\left(\int_{\mathbb{R}_+^n} f^p v\right)^{\frac{1}{p}}}$$

are given, where the supremum is taken over all such functions f.

The same type of results over the cone of radially decreasing functions, but in this case for general weight functions will be also considered.

Applications of these results in connection with boundedness of Hardy type operators will be pointed out.

MIHÁLY BESSENYEI AND ZSOLT PÁLES

Higher-order Generalizations of Hadamard's Inequality

ABSTRACT

Let $I \subset \mathbb{R}$ be a proper interval. A function $f : I \to \mathbb{R}$ is said to be n-monotone, if

$$(-1)^n \begin{vmatrix} f(x_0) & \dots & f(x_n) \\ 1 & \dots & 1 \\ x_0 & \dots & x_n \\ \vdots & & \vdots \\ x_0^{n-1} & \dots & x_n^{n-1} \end{vmatrix} \geq 0,$$

whenever $x_0 < \ldots < x_n$, $x_0, \ldots, x_n \in I$. Obviously, a function f is 2-monotone if and only if it is convex. According to Hadamard's classical result, the inequalities

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}$$

hold for any convex, i.e., for 2-monotone function $f : [a,b] \to \mathbb{R}$. Our goal is to generalize this result for n-monotone functions and present some applications. For instance, if the function $f : [a,b] \to \mathbb{R}$ is supposed to be 3-monotone, one can deduce that

$$\frac{f(a)+3f\left(\frac{a+2b}{3}\right)}{4} \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(b)+3f\left(\frac{2a+b}{3}\right)}{4}.$$

MALCOLM BROWN

Everitt's HELP Inequality and Its Successors

ABSTRACT

In 1971 Everitt introduced the inequality

$$\left(\int_a^{\to b}(pf'^2+qf^2)dx\right)^2 \le K\int_a^b wf^2dx\int_a^b w\big(w^{-1}(-(pf')'+qf)\big)^2dx$$

for functions f from

$$\{f:[a,b)\to \mathrm{R}f, pf'\in AC_{loc}[a,b)f, w^{-1}(-(pf')'+qf)\in L^2(a,b;w)\}.$$

He showed that the validity of the inequality, (ie. finite K) and cases of equality were dependent on the spectral properties of the operator defined from $1/w(-(pf')'+qf))$ in the Hilbert space $L^2_w[a,b)$.

The talk will explore the class of inequalities

$$A^2(f)\le KB(f)C(f)$$

which have associated with them a self-adjoint operator acting in a domain of a Hilbert space. This class will generate examples of inequalities between members of infinite sequences and also inequalities between a function and its higher order derivatives.

R.C. BROWN

Some Separation Criteria and Inequalities Associated with Linear Differential and Partial Differential Operators

ABSTRACT

In a series of remarkable papers between 1971 and 1977 W. N. Everitt and M. Giertz determined several sufficient conditions for *separation*, i.e., given a second order symmetric differential operator $M_w[y]=w^{-1}(-(py')'+qy)$ defined in $L^2(w;I)$, $I=(a,b)$ with one or both end-points singular, the property that $y, M_w[y]\in L^2(w;I)\implies w^{-1}qy\in L^2(w;I)$. Here we trace some recent developments concerning this problem and its generalizations to the higher order case and classes of partial differential operators due to the Russian school, D. B. Hinton, and the author.

Several new criteria for separation are given. Some of these are quite different than those of Everitt and Giertz; others are natural generalizations of their results, and some can be extended so that they yield separation for partial differential operators. We also point out a separation problem for non-selfadjoint operators due to Landau in 1929 and study the connection between separation and other spectral properties of M_w and associated operators.

This paper is published online at `http://jipam.vu.edu.au/v4n3/130_02.html`.

CONSTANTIN BUŞE

A Landau-Kallman-Rota's Type Inequality For Evolution Semigroups

ABSTRACT

Let X be a complex Banach space, $\mathbb{R}_+$ the set of all non-negative real numbers and let $\mathbb{J}$ be either, or $\mathbb{R}$ or $\mathbb{R}_+$. The Banach space of all X-valued, bounded and uniformly continuous functions on $\mathbb{J}$ will be denoted by $BUC(\mathbb{J}, X)$ and the Banach space of all X-valued, almost periodic functions on $\mathbb{J}$ will be denoted by $AP(\mathbb{J}, X)$. $C_0(\mathbb{R}_+, X)$ is the subspace of $BUC(\mathbb{R}_+, X)$ consisting of all functions for which $\lim_{t\to\infty} f(t) = 0$ and $C_{00}(\mathbb{R}_+, X)$ is the subspace of $C_0(\mathbb{R}_+, X)$ consisting of all functions f for which $f(0) = 0$. It is known that $AP(\mathbb{J}, X)$ is the smallest closed subspace of $BUC(\mathbb{J}, X)$ containing functions of the form: $t \mapsto e^{i\mu t}x; \mu \in \mathbb{R}, x \in X, t \in \mathbb{J}$. The set of all X-valued functions on $\mathbb{R}_+$ for which there exist $t_f \geq 0$ and F_f in $AP(\mathbb{R}, X)$ such that $f(t) = 0$ if $t \in [0, t_f]$ and $f(t) = F_f(t)$ if $t \geq t_f$ will be denoted by $\mathcal{A}_0(\mathbb{R}_+, X)$. The smallest closed subspace of $BUC(\mathbb{R}_+, X)$ which contains $\mathcal{A}_0(\mathbb{R}_+, X)$ will be denoted by $AP_0(\mathbb{R}_+, X)$. $AAP_0(\mathbb{R}_+, X)$ denotes here the subspace of $BUC(\mathbb{R}_+, X)$ consisting of all functions $h : \mathbb{R}_+ \to X$ for which there exist $t_f \geq 0$ and $F_f \in AP(\mathbb{R}_+, X)$ such that $h = f + g$ and $(f + g)(0) = 0$. Let $\mathcal{X}$ one of the following spaces: $C_{00}(\mathbb{R}_+, X)$, $AP_0(\mathbb{R}_+, X)$, $AAP_0(\mathbb{R}_+, X)$. The main result can be formulated as follows:

Theorem. Let f be a function belonging to $\mathcal{X}$ and $\mathcal{U} = \{U(t, s) : t \geq s \geq 0\}$ be an 1-periodic evolution family of bounded linear operators acting on X. If $\mathcal{U}$ is bounded (i.e., $\sup_{t\geq s\geq 0} ||U(t, s)|| = M < \infty$) and the functions $g(\cdot) := \int_0^{\cdot} U(\cdot, s)f(s)ds$ and $h(\cdot) := \int_0^{\cdot} (\cdot - s)U(\cdot, s)f(s)ds$ belong to $\mathcal{X}$ then $||g||_{\mathcal{X}} \leq 4M^2||f||_{\mathcal{X}}||h||_{\mathcal{X}}$.

PIETRO CERONE

On Some Results Involving The Čebyšev Functional and Its Generalizations

ABSTRACT

Recent results involving bounds of the Čebyšev functional to include means over different intervals are extended to a measurable space setting. Sharp bounds are obtained for the resulting expressions of the generalized Čebyšev functionals where the means are over different measurable sets.

This paper is published online at `http://jipam.vu.edu.au/v4n3/124_02.html`.

PÉTER CZINDER AND ZSOLT PÁLES

Minkowski-type Inequalities For Two Variable Homogeneous Means

ABSTRACT

There is an extensive literature on the Minkowski-type inequality

$$M_{a,b}(x_1 + y_1, x_2 + y_2) \leq M_{a,b}(x_1, x_2) + M_{a,b}(y_1, y_2) \tag{1}$$

and its reverse, where $M_{a,b}$ stands for the Gini mean

$$G_{a,b}(x_1, x_2) = \left(\frac{x_1^a + x_2^a}{x_1^b + x_2^b}\right)^{\frac{1}{a-b}} \qquad (a - b \neq 0),$$

or for the Stolarsky mean

$$S_{a,b}(x_1,x_2)=\left(\frac{x_1^a-x_2^a}{a}\frac{b}{x_1^b-x_2^b}\right)^{\frac{1}{a-b}}\qquad (\,ab(a-b)\neq 0\,)$$

with positive variables. (These mean values can be extended for any real parameters a and b.)

A possibility to generalize (1) is that each appearance of $M_{a,b}$ is replaced by a different mean, that is, we ask for necessary and/or sufficient conditions such that

$$M_{a_0,b_0}(x_1+y_1,x_2+y_2)\leq M_{a_1,b_1}(x_1,x_2)+M_{a_2,b_2}(y_1,y_2)$$

or the reverse inequality be valid for all positive x_1, x_2, y_1, y_2.

We summarize our main results obtained in this field.

ZOLTÁN DARÓCZY AND ZSOLT PÁLES

On The Comparison Problem For a Class of Mean Values

ABSTRACT

Let $I\subseteq\mathbb{R}$ be a non-empty open interval. The function $M:I^2\to I$ is called a strict pre-mean on I if

(i) $M(x,x)=x$ for all $x\in I$ and
(ii) $\min\{x,y\}<M(x,y)<\max\{x,y\}$ if $x,y\in I$ and $x\neq y$.

The function $M:I^2\to I$ is called a strict mean on I if M is a strict pre-mean on I and M is continuous on I^2.

Denote by $\mathcal{CM}(I)$ the class of continuous and strictly monotone real functions defined on the interval I.

Let $L:I^2\to I$ be a fixed strict pre-mean and $p,q\in]0,1]$. We call $M:I^2\to I$ an L-conjugated mean of order (p,q) on I if there exists a $\varphi\in\mathcal{CM}(I)$ such that

$$M(x,y)=\varphi^{-1}[p\varphi(x)+q\varphi(y)+(1-p-q)\varphi(L(x,y)]=:L_\varphi^{(p,q)}(x,y)$$

for all $x,y\in I$.

In the present paper we treat the problem of comparison (and equality of) L-conjugated means of order (p,q), that is, the inequality $L_\varphi^{(p,q)}(x,y)\leq L_\psi^{(p,q)}(x,y)$ where $x,y\in I$; $\varphi,\psi\in\mathcal{CM}(I)$ and $p,q\in]0,1]$. Our results include several classical cases such as the weighted quasi-arithmetic and the conjugated arithmetic means.

SILVESTRU SEVER DRAGOMIR

New Inequalities of Grüss Type For Riemann-Stieltjes Integral

ABSTRACT

New inequalities of Grüss type for Riemann-Stieltjes integral and applications for different weights are given.

This paper is published online at `http://rgmia.vu.edu.au/v5n4.html` as Article 3.

A.M. FINK

Best Possible Andersson Inequalities

ABSTRACT

Andersson has shown that if f_i are convex and increasing with $f_i(0) = 0$, then

$$\int_0^1 (f_1 \cdots f_n)dx \geq \frac{2^n}{n+1} \left(\int_0^1 f_1(x)dx \right) \cdots \left(\int_0^1 f_n(x)dx \right).$$

We turn this into a "best possible inequality" which cannot be generalized by expanding the set of functions f_i and the measures.

This paper is published online at `http://jipam.vu.edu.au/v4n3/106_02.html`.

ROMAN GER

Stability of ψ−Additive Mappings and Orlicz Δ_2-Condition

ABSTRACT

We deal with a functional inequality of the form

$$\|f(x+y) - f(x) - f(y)\| \leq \varphi(\|x\|) + \psi(\|y\|)\,, \tag{1}$$

showing, among others, that given two selfmappings φ, ψ of the halfline $[0, \infty)$ enjoying the celebrated Orlicz Δ_2 conditions:

$$\varphi(2t) \leq k\varphi(t), \qquad \psi(2t) \leq \ell\psi(t)$$

for all $t \in [0, \infty)$, with some constants $k, \ell \in [0, 2)$, for every map f between a normed linear space $(X, \|\cdot\|)$ and a Banach space $(Y, \|\cdot\|)$ satisfying inequality (1) there exists exactly one additive map $a : X \longrightarrow Y$ such that

$$\|f(x) - a(x)\| \leq \frac{1}{2-k}\varphi(\|x\|) + \frac{1}{2-\ell}\psi(\|x\|)$$

for all $x \in X$. This generalizes (in several simultaneous directions) a result of G. Isac & Th. M. Rassias (*J. Approx. Theory*, **72** (1993), 131-137); see also the monograph *Stability of functional equations in several variables* by Donald H. Hyers, George Isac and Themistocles M. Rassias (Birkhäuser, Boston-Basel-Berlin, 1998, Theorem 2.4).

ATTILA GILÁNYI AND ZSOLT PÁLES

On Convex Functions of Higher Order

ABSTRACT

Higher-order convexity properties of real functions are characterized in terms of Dinghas-type derivatives. The main tool used is a mean value inequality for those derivatives.

ATTILA HÁZY AND ZSOLT PÁLES

On Approximately Midconvex Functions

ABSTRACT

A real valued function f defined on an open convex set D is called (ε,δ)-midconvex if it satisfies

$$f\left(\frac{x+y}{2}\right) \le \frac{f(x)+f(y)}{2} + \varepsilon|x-y| + \delta \quad \text{for } x,y \in D.$$

The main result states that if f is locally bounded from above at a point of D and is (ε,δ)-midconvex then it satisfies the convexity-type inequality

$$f\big(\lambda x + (1-\lambda)y\big) \le \lambda f(x) + (1-\lambda)f(y) + 2\delta + 2\varepsilon\varphi(\lambda)|x-y| \quad \text{for } x,y \in D,\ \lambda\in[0,1],$$

where $\varphi : [0,1] \to \mathbb{R}$ is a continuous function satisfying

$$\max\big(-\lambda\log_2\lambda,\ -(1-\lambda)\log_2(1-\lambda)\big) \le \varphi(\lambda) \le c\max\big(-\lambda\log_2\lambda,\ -(1-\lambda)\log_2(1-\lambda)\big)$$

with $1 < c < 1.4$. The particular case $\varepsilon = 0$ of this result is due to Nikodem and Ng [1], the specialization $\varepsilon = \delta = 0$ yields the theorem of Bernstein and Doetsch [2].

REFERENCES

[1] C.T. NG AND K. NIKODEM, On approximately convex functions, *Proc. Amer. Math. Soc.*, **118**(1) (1993), 103–108.

[2] F. BERNSTEIN AND G. DOETSCH, Zur Theorie der konvexen Funktionen, *Math. Annalen*, **76** (1915), 514–526.

GERD HERZOG

Semicontinuous Solutions of Systems of Functional Equations

ABSTRACT

For a metric space Ω, and functions $F : \Omega \times \mathbb{R}^{(1+m)n} \to \mathbb{R}^n$ and $g_j : \Omega \to \Omega$ the following functional equation is considered:

$$F(\omega, u(\omega), u(g_1(\omega)), \ldots, u(g_m(\omega))) = 0.$$

We assume that $\mathbb{R}^n$ is ordered by a cone and prove the existence of upper and lower semicontinuous solutions under monotonicity and quasimonotonicity assumptions on F. For example, the results can be applied to systems of elliptic difference equations.

JÓZSEF KOLUMBÁN

Generalization of Ky Fan's Minimax Inequality

ABSTRACT

We give a generalization of the following useful theorem:

Theorem. (Ky Fan, 1972) Let X be a nonempty, convex, compact subset of a Hausdorff topological vector space E and let $f : X \times X \to \mathbb{R}$ such that

$$\forall y \in X, \quad f(\cdot, y) : X \to \mathbb{R} \text{ is upper semicontinuous},$$

$$\forall x \in X, \quad f(x, \cdot) : X \to \mathbb{R} \text{ is quasiconvex}$$

and

$$\forall x \in X, \quad f(x, x) \geq 0.$$

Then there exists an element $x_0 \in X$, such that $f(x_0, y) \geq 0$ for each $y \in X$.

ALOIS KUFNER

Hardy's Inequality and Compact Imbeddings

ABSTRACT

It is well known that the value $p^* = \frac{Np}{N-p}$ is the critical value of the imbedding of $W^{1,p}(\Omega)$ into $L^q(\Omega)$, $\Omega \subset \mathbb{R}^N$. In the talk, an analogue of this critical value for imbeddings between *weighted* spaces will be determined. More precisely, a value $p^* = p^*(p, q, u, v)$ will be determined such that the Hardy inequality

$$\left(\int_a^b |f(t)|^q u(t)\, dt \right)^{\frac{1}{q}} \leq C \left(\int_a^b |f'(t)|^p v(t)\, dt \right)^{\frac{1}{p}}$$

with $1 < p \leq \infty$ and $f(b) = 0$ expresses an imbedding which is compact for $q < p^*$ and does not hold for $q > p^*$.

Applications to the spectral analysis of certain nonlinear differential operators will be mentioned.

ROLAND LEMMERT AND GERD HERZOG

Second Order Elliptic Differential Inequalities in Banach Spaces

ABSTRACT

We derive monotonicity results for solutions of partial differential inequalities (of elliptic type) in ordered normed spaces with respect to the boundary values. As a consequence, we get an existence theorem for the Dirichlet boundary value problem by means of a variant of Tarski's Fixed Point Theorem.

LÁSZLÓ LOSONCZI

Sub- and Superadditive Integral Means

ABSTRACT

If $f : I \to \mathbb{R}$ is continuous and strictly monotonic on the interval I then for every $x_1, x_2 \in I,\ x_1 < x_2$ there is a point $s \in]x_1, x_2[$ such that

$$f(s) = \frac{\int_{x_1}^{x_2} f(u)\,du}{x_2 - x_1} \quad \text{thus} \quad s = f^{-1}\left(\frac{\int_{x_1}^{x_2} f(u)\,du}{x_2 - x_1}\right).$$

This number s is called the *integral f-mean of* x_1 and x_2 and denoted by $I_f(x_1, x_2)$. Clearly, (requiring I_f to have the mean property or be continuous) we have for equal arguments $I_f(x, x) = x \quad (x \in I)$. By the help of divided differences I_f can easily be defined for more than two variables.

Here we completely characterize the sub- and superadditive integral means on suitable intervals I, that is we give necessary and sufficient conditions for the inequality

$$I_f(x_1 + y_1, \dots, x_n + y_n) \le I_f(x_1, \dots, x_n) + I_f(y_1, \dots, y_n) \quad (x_i, y_i \in I)$$

and its reverse.

RAM N. MOHAPATRA

Grüss Type Inequalities and Error of Best Approximation

ABSTRACT

In this paper we consider recent results on Grüss type inequalities and provide a connection between a Grüss type inequality and the error of best approximation. We also consider unification of discrete and continuous Grüss type inequalities.

CONSTANTIN P. NICULESCU

Noncommutative Extensions of The Poincaré Recurrence Theorem

ABSTRACT

Recurrence was introduced by H. Poincaré in connection with his study on Celestial Mechanics and refers to the property of an orbit to come arbitrarily close to positions already occupied. More precisely, *if T is a measure-preserving transformation of a probability space* (Ω, Σ, μ), *then for every* $A \in \Sigma$ *with* $\mu(A) > 0$ *there exists an* $n \in \mathbf{N}^\star$ *such that* $\mu(T^{-n}A \cap A) > 0$.

In his famous solution to the Szemerédi theorem, H. Furstenberg [1], [2] was led to formulate the following multiple recurrence theorem which extends the Poincaré result: *For every measure-preserving transformation T of a probability space* (Ω, Σ, μ), *every* $A \in \Sigma$ *with* $\mu(A) > 0$ *and every* $k \in \mathbf{N}^\star$,

$$\liminf_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \mu(A \cap T^{-n}A \cap \cdots \cap T^{-kn}A) > 0. \tag{1}$$

The aim of our talk is to discuss the formula (1) in the context of C^*-dynamical systems. Details appear in [4].

REFERENCES

[1] H. FURSTENBERG, *Recurrence in Ergodic Theory and Combinatorial Number Theory,* Princeton Univ. Press, New Jersey, 1981.

[2] H. FURSTENBERG, Y. KATZNELSON AND D. ORNSTEIN, The ergodic theoretical proof of Szemerédi theorem, *Bull. Amer. Math. Soc.* (New Series), **7** (1982), 527–552.

[3] A. Y. KHINTCHINE, Eine Verschärfung des Poincaré'schen Wiederkechrsatzes, *Comp. Math.*, **1** (1934), 177–179.

[4] C. P. NICULESCU, A. STRÖH AND L. ZSIDÓ, Noncommutative extensions of classical and multiple recurrence theorems. Preprint, 2002.

KAZIMIERZ NIKODEM, MIROSŁAW ADAMEK AND ZSOLT PÁLES

On (K, λ)-Convex Set-valued Maps

ABSTRACT

Let D be a convex set, $\lambda : D^2 \to (0, 1)$ be a given function and K be a convex cone in a vector space Y. A set-valued map $F : D \to n(Y)$ is called (K, λ)-*convex* if

$$\lambda(x, y)F(x) + (1 - \lambda(x, y))F(y) \subset F\big(\lambda(x, y)x + (1 - \lambda(x, y))y\big) + K$$

for all $x, y \in D$. The map F is said to be K-*convex* if

$$tF(x) + (1 - t)F(y) \subset F(tx + (1 - t)y) + K, \qquad x, y \in D,\ t \in [0, 1].$$

Conditions under which (K, λ)-convex set-valued maps are K-convex are discussed. In particular, the following generalizations of the theorems of Bernstein–Doetsch and Sierpinski are given.

Theorem 1. *Let $D \subset \mathbb{R}^n$ be an open convex set, $\lambda : D^2 \to (0, 1)$ be a function continuous in each variable, Y be a locally convex space and K be a closed convex cone in Y. If a set-valued map $F : D \to c(Y)$ is (K, λ)-convex and locally K-upper bounded at a point of D, then it is K-convex.*

Theorem 2. *Let Y, K, and D be such as in Theorem 1 and $\lambda : D^2 \to (0, 1)$ be a continuously differentiable function. If a set-valued map $F : D \to c(Y)$ is (K, λ)-convex and Lebesgue measurable, then it is also K-convex.*

ZSOLT PÁLES

Comparison of Generalized Quasiarithmetic Means

ABSTRACT

If $f_1, \dots, f_k$ (where $k \geq 2$) are strictly increasing continuous functions defined on an open interval I, then the k-variable function

$$\mathcal{M}_{f_1,\dots,f_k}(x_1, \dots, x_k) := \big(f_1 + \cdots + f_k\big)^{-1}\Big(f_1(x_1) + \cdots + f_k(x_k)\Big)$$

defines a k-variable mean on I. In the case $f_1 = \cdots = f_k = f$, the resulting mean is a so-called *quasiarithmetic mean*, therefore, functions of the form $\mathcal{M}_{f_1,\dots,f_k}$ can be considered as generalizations of quasiarithmetic means.

In our main results, we offer necessary and sufficient conditions on $f_1, \ldots, f_k$ and $g_1, \ldots, g_k$ in order that the comparison inequality

$$\mathcal{M}_{f_1,\ldots,f_k}(x_1, \ldots, x_k) \le \mathcal{M}_{g_1,\ldots,g_k}(x_1, \ldots, x_k)$$

be valid for all $x_1, \ldots, x_k \in I$.

In another result, a characterization of generalized quasiarithmetic means in terms of regularity properties and functional equations is also presented.

CHARLES PEARCE

On The Relative Values of Means

ABSTRACT

A consequence of the AGH inequality for a pair of distinct positive numbers is that the AG gap exceeds the GH gap. Scott has shown that this does not extend to $n > 2$ numbers and gives a counterexample for $n = 4$.

The question of what happens for general n has been addressed by Lord and by Pečarić and the present author, who showed that a number of analytical and statistical issues are involved. These studies left further open questions, including explicit representations for the functional forms of certain extrema.

The present study proceeds with these and related questions.

This paper is published online at `http://jipam.vu.edu.au/v4n3/008_03.html`.

LARS-ERIK PERSSON AND ALOIS KUFNER

Weighted Inequalities of Hardy Type

ABSTRACT

I briefly present some historical remarks and recent developments of some Hardy type inequalities and their limit (Carleman–Knopp type) inequalities. Some open questions are mentioned.

REFERENCES

[1] A. KUFNER AND L.E. PERSSON, *Weighted Inequalities of Hardy Type*, World Scientific (to appear).

MICHAEL PLUM, H. BEHNKE, U. MERTINS, AND Ch. WIENERS

Eigenvalue Enclosures Via Domain Decomposition

ABSTRACT

A computer-assisted method will be presented which provides eigenvalue enclosures for the Laplacian with Neumann boundary conditions on a domain $\Omega \subset \mathbb{R}^2$. While upper eigenvalue bounds are easily accessible via the Rayleigh-Ritz method, lower bounds require much more effort. On the one hand, we propose an appropriate setting of Goerisch's method for this purpose; on the other hand, we introduce a new kind of of homotopy to obtain the spectral a priori information needed for Goerisch's method (as for any other method providing lower eigenvalue bounds). This homotopy is based on a decomposition of Ω into simpler subdomains and their "continuous" rejoining. As examples, we consider a bounded domain Ω with two "holes", and

an acoustic waveguide, where Ω is an infinite strip minus some compact obstacle. Moreover, we discuss an application to the (nonlinear) Gelfand equation.

SABUROU SAITOH, V.K. TUAN AND M. YAMAMOTO

Reverse Convolution Inequalities and Applications

ABSTRACT

Reverse convolution norm inequalities and their applications to various inverse problems are introduced which are obtained in the references.

At first, for some general principle, we show that we can introduce various operators in Hilbert spaces through by linear and nonlinear transforms and we can obtain various norm inequalities, by minimum principle.

From a special case, we obtain weighted L_p norm inequalities in convolutions and we can show many concrete applications to forward problems.

On the basis of the elementary proof in the weighted L_p convolution inequalities, by using reverse Hölder inequalities, we can obtain reverse weighted L_p convolution inequalities and concrete applications to various inverse problems.

By elementary means, we can obtain reverse Hölder inequalities for weak conditions which have very important applications and related reverse weighted L_p convolution inequalities. We show concrete applications to inverse heat source problems.

We recently found that the theory of reproducing kernels is applied basically in Statistical Learning Theory. See, for example, F. Cucker and S. Smale, On the mathematical foundations of learning, *Bull. Amer. Math. Soc.*, **39** (2001), 1–49. When we have time, I would like to present our recent convergence rate estimates and related norm inequalities whose types are appeared in Statistical Learning Theory.

REFERENCES

[1] S. SAITOH, Various operators in Hilbert space introduced by transforms, *Internat. J. Appl. Math.*, **1** (1999), 111–126.

[2] S. SAITOH, Weighted L_p-norm inequalities in convolutions, *Survey on Classical Inequalities*, Kluwer Acad. Publ., 2000, pp. 225–234.

[3] S. SAITOH, V.K. TUAN AND M. YAMAMOTO, Reverse weighted L_p-norm inequalities in convolutions and stability in inverse problems, *J. Inequal. Pure Appl. Math.*, **1**(1) (2000), Art. 7. [ONLINE http://jipam.vu.edu.au/v1n1/018_99.html]

[4] S. SAITOH, V.K. TUAN AND M. YAMAMOTO, Reverse convolution inequalities and applications to inverse heat source problems, (in preparation).

[5] S. SAITOH, Some general approximation error and convergence rate estimates in statistical learning theory (in preparation).

This paper is published online at http://jipam.vu.edu.au/v4n3/138_02.html.

ANTHONY SOFO

An Integral Approximation in Three Variables

ABSTRACT

In this presentation I will describe a method of approximating an integral in three independent variables. The Ostrowski type inequality is established by the use of Peano kernels and improves a result given by Pachpatte.

This paper is published online at http://jipam.vu.edu.au/v4n3/125_02.html.

SILKE STAPELKAMP

The Brézis-Nirenberg Problem on $\mathbb{H}^n$ and Sobolev Inequalities

ABSTRACT

We consider the equation $\Delta_{\mathbb{H}^n} u + u^{\frac{2n}{n-2}-1} + \lambda u = 0$ in a domain D' in hyperbolic space $\mathbb{H}^n$, $n \geq 3$ with Dirichlet boundary conditions. For different values of λ we search for positive solutions $u \in \mathbb{H}_0^{1,2}(D')$.

Existence holds for $\lambda^* < \lambda < \lambda_1$, where we can compute the value of λ^* exactly if D' is a geodesic ball. For this result we should derive some Sobolev type inequalities.

The existence result will be used to develop some results for more general equations of the form $\Delta_\rho u + u^{\frac{2n}{n-2}-1} + \lambda u = 0$. Here $\Delta_\rho = \rho^{-n}\nabla(\rho^{n-2}\nabla u)$ denotes the Laplace-Beltrami operator corresponding to the conformal metric $ds = \rho(x)|dx|$.

It turns out that if $\rho = \rho_1\rho_2$ you can find an inequality for ρ_1 and ρ_2 that gives you an existence result.

JACEK TABOR

On Localized Derivatives and Differential Inclusions

ABSTRACT

It often happens that a function is not differentiable at a given point, but to some extent it seems that the "nonexistent derivative" has some properties.

Let us look for example at the function $x(t) := |t|$. Then x is not differentiable at zero, but there exist left and right derivatives, which equal to -1 and 1, respectively. Thus in a certain sense, which we make formal below, we may say that the derivative belongs to the set $\{-1, 1\}$. Let us now consider another example. As we now there exist a Banach space X and a lipschitz with constant 1 function which is nowhere differentiable. This suggest that the "nonexistent derivative" of this function belongs to the unit ball.

The above ideas lead us to the following definition. We assume that I is a subinterval of the real line and that X is a Banach space.

Definition. Let $x : I \to X$ and let V be a closed subset of X. We say that the derivative of x at t is localized in V, which we write $Dx(t) \Subset V$, if

$$\lim_{h\to 0} d\left(\frac{x(t+h)-x(t)}{h}; V\right) = 0,$$

where $d(a; B)$ denotes the distance of the point a from the set B

One of the main results we prove is:

Theorem. Let $x : I \to X$ and let V be a closed convex subset of X. We assume that $Dx(t) \Subset V$ for $t \in I$. Then

$$\frac{x(q) - x(p)}{q - p} \in V \quad \text{for } p, q \in I.$$

As a direct corollary we obtain a local characterization of increasing functions.

Corollary. Let $x : I \to \mathbb{R}$. Then x is increasing iff $Dx(t) \Subset \mathbb{R}_+$ for $t \in I$.

WOLFGANG WALTER

Infinite Quasimonotone Systems of ODEs With Applications to Stochastic Processes

ABSTRACT

We deal with the initial value problem for countably infinite linear systems of ordinary differential equations of the form $y'(t) = A(t)y(t)$ where $A(t) = (a_{ij}(t) : i, j \geq 1)$ is an infinite, essentially positive matrix, i.e., $a_{ij}(t) \geq 0$ for $i \neq j$. The main novelty of our approach is the systematic use of a classical theorem on sub- and supersolutions for finite linear systems which leads easily to the existence of a unique nonnegative minimal solution and its properties. Application to generalized stochastic birth and death processes leads to conditions for honest and dishonest probability distributions. The results hold for L^1-coefficients. Our method extends to nonlinear infinite systems of quasimonotone type.

ANNA WEDESTIG

Some New Hardy Type Inequalities and Their Limiting Carleman-Knopp Type Inequalities

ABSTRACT

New necessary and sufficient conditions for the weighted Hardy's inequality is proved. The corresponding limiting Carleman-Knopp inequality is also proved and also the corresponding limiting result in two dimensions is pointed out.

3. PROBLEMS AND REMARKS

3.1. Problem.

Investigating the stability properties of convexity, Hyers and Ulam [1] obtained the following result:

Theorem 1. *Let $D \subset \mathbb{R}^n$ be a convex set. Then there exists a constant c_n such that if a function $f : D \to \mathbb{R}$ is ε-convex on D, i.e., if it satisfies*

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) + \varepsilon \qquad (x, y \in D,\ t \in [0,1])$$

(where ε is a nonnegative constant), then it is of the form $f = g + h$, where g is a convex function and h is a bounded function with $\|h\| \leq c_n\varepsilon$.

An analogous result was obtained for (ε, δ)-convex real functions in [2]:

Theorem 2. *Let $D \subset \mathbb{R}$ be an open interval. Assume that $f : D \to \mathbb{R}$ is (ε, δ)-convex on D, i.e., if it satisfies*

$$(1)\quad f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) + \varepsilon + \delta t(1-t)\|x-y\| \qquad (x, y \in D,\ t \in [0,1])$$

(where ε and δ are nonnegative constants), then it is of the form $f = g + h + \ell$, where g is a convex function and h is a bounded function with $\|h\| \leq \varepsilon/2$ and ℓ is a Lipschitz function with Lipschitz modulus not greater than δ.

Problem. Let $D \subset \mathbb{R}^n$ be an open convex set. Do there exist constants c_n and d_n so that whenever a function $f : D \to \mathbb{R}$ satisfies (1) then it must be of the form $f = g + h + \ell$, where g is convex, h is bounded with $\|h\| \leq c_n\varepsilon$, and ℓ is Lipschitz with Lipschitz modulus not greater than $d_n\delta$.

REFERENCES

[1] D.H. HYERS AND S.M. ULAM, Approximately convex functions, *Proc. Amer. Math. Soc.*, **3** (1952), 821–828.

[2] Zs. PÁLES, On approximately convex functions, *Proc. Amer. Math. Soc.*, **131** (2003), 243–252.

Zsolt Páles.

3.2. **Problem and Remark.**

László Fuchs and I, working then as now mainly in algebra and analysis, respectively, wrote 55 years ago a paper in geometry, that Fuchs considers a "folly of his youth" - I don't - and that was called "beautiful" by L. E. J. Brouwer (maybe because it contained no proof by contradiction) and published in his journal Compositio Mathematica **8** (1950), 61–67. The result was as follows.

Inscribe a convex (not necessarily regular) n-gon into a circle and by drawing tangents at the vertices, also a circumscribed n-gon. The sum of areas of these two polygons has an absolute (from n independent) minimum at the pair of squares. The proof was analytic.

The problem has been raised repeatedly of finding a geometric proof. No such proof has been found up to now and the problem seems to be rather difficult.

The above result implies that there is no minimum for n-gon pairs with fixed $n \geq 5$. Paul Erdős asked in 1983 whether the pair of regular triangles yields the minimal area-sum for inscribed and circumscribed triangles. The answer is yes, as proved by Jürg Rätz and by me independently. Both of us used analytic methods.

Of course, again a search for a geometric proof was launched immediately. This proved easier to find. With P. Schöpf (Univ. Graz, Austria) we found an essentially geometric proof in 2000 and it appeared recently in Praxis der Mathematik **4** (2002), 133–135.

János Aczél.

3.3. **Problem.**

Let f be an increasing continuous function mapping a unit interval $[0,1]$ onto itself. Let $n \in \mathbb{N}$.

Definition. Let V be a subset of a vector space X. We say that V is (n,f)-convex if for every $\alpha_1,\dots,\alpha_n \in [0,1]$ such that $f(\alpha_1)+\dots+f(\alpha_n)=1$ and every $v_1,\dots,v_n \in V$ we have $\alpha_1 v_1+\dots+\alpha_n v_n \in V$.

Problem. Find (characterize) all f such that every $(2,f)$-convex set is (n,f)-convex for arbitrary $n \in \mathbb{N}$.

Jacek Tabor.

3.4. **Problem.**

(i) The implication

$$W_0^{1,p}(\Omega) \to L^q(\Omega) \;\Rightarrow\; W_0^{1,p}(\Omega) \to L^{\hat{q}}(\Omega) \qquad \text{for } \hat{q} < q$$

follows easily by Hölder's inequality, provided

(1) $$\text{meas}\,\Omega < \infty.$$

(ii) For weighted Sobolev spaces (a, b weight functions), the implication

(2) $$W_0^{1,p}(\Omega;a) \to L^q(\Omega;b) \;\Rightarrow\; W_0^{1,p}(\Omega;a) \to L^{\hat{q}}(\Omega;b) \qquad \text{for } \hat{q} < q$$

follows easily by Hölder's inequality, provided

(3) $$b \in L^1(\Omega).$$

Notice that (1) means (3) for $b \equiv 1$.

Problem. Is (3) not only sufficient, but also necessary for the implication (2)?

Alois Kufner.

3.5. **Problem.**

A classic result due to Opial [6] says that the best constant K of the inequality

(1) $$\int_0^1 |yy'|\,dx \le K \int_0^1 (y')^2\,dx, \qquad y(0)=y(1)=0$$

for all real functions $y \in \mathcal{D}$, where

$$\mathcal{D} = \{y : y \text{ is absolutely continuous and } y' \in L^2(0,1)\}$$

is $\frac{1}{4}$ and that the extremals s_c are of the form

$$s_c(x) = \begin{cases} cx & \text{if } 0 \le x \le \frac{1}{2}, \\ c(1-x) & \text{if } \frac{1}{2} < x \le 1, \end{cases}$$

where c is a constant.

We note that the existence of an inequality of the form (1) is quite easy to prove. For, if we apply the Cauchy-Schwarz and a form of the Wirtinger inequality [4, p. 67] we see that

$$\int_0^1 |yy'|\,dx \le \left(\int_0^1 y^2\,dx\right)^{\frac{1}{2}} \left(\int_0^1 (y')^2\,dx\right)^{\frac{1}{2}} \le \frac{1}{\pi}\int_0^1 (y')^2\,dx.$$

The nontrivial part of (1) is the determination of the least value of K and the characterization of the extremals. The original proof of Opial [6] assumed that $y > 0$. This restriction was eliminated by Olech [5]. At least six proofs are known and may be found in [1], [4]. In order to get a feeling for the subtleties involved in Opial's inequality we give a proof which is close to Olech's.

Proof. For $y \in \mathcal{D}$ satisfying the boundary conditions of (1) let $p \in (0,1)$ satisfy

$$\int_0^p |y'|\,dx = \int_p^1 |y'|\,dx.$$

Define

$$Y(x) = \begin{cases} \int_0^x |y'| & \text{if } x \in [0,p] \\ \int_x^1 |y'| & \text{if } x \in (p,1]. \end{cases}$$

Evidently, $Y(0) = Y(1) = 0$, $|y| \le Y$, $Y \in \mathcal{D}$, and

$$K^{-1} \le \frac{\int_0^1 |Y'|^2\,dx}{\int_0^1 |YY'|\,dx} \le \frac{\int_0^1 |y'|^2\,dx}{\int_0^1 |yy'|\,dx}.$$

Thus an extremal of (1) (if any) will be found among the class $\mathcal{D}' \subset \mathcal{D}$ consisting of those y satisfying $y(0) = y(1) = 0$ which are nondecreasing on $(0,p]$, nonincreasing on $(p,1]$, and such that $y(p) = 1$. Then $\int_0^1 |yy'|\,dx = 1$, and

$$K^{-1} = \inf_{y \in \mathcal{D}'} \int_0^1 |y'|^2\,dx.$$

The extremal is evidently a linear spline with a unique knot at p. Moreover,

$$K^{-1} = \inf_{p \in (0,1)} \left(\frac{1}{p} + \frac{1}{1-p} \right) = 4.$$

By a variation of the above proof [3] one can show that

$$\int_0^1 |yy'|\,dx \le \frac{1}{4} \int_0^1 (y')^2\,dx \qquad \text{whenever } y(0) + y(1) = 0$$

for $y \in \mathcal{D}$. □

Now consider the inequality

(2) $$\int_0^1 |yy'|\,dx \le K \int_0^1 (y')^2\,dx \qquad \text{whenever } \int_0^1 y\,dx = 0,$$

where $y \in \mathcal{D}$. An upper bound for K in (2) is also $\frac{1}{\pi}$. If we set $y = x - \frac{1}{2}$ a calculation shows that a lower bound on $K = \frac{1}{4}$.

Conjecture. The best value of K in (2) is also $\frac{1}{4}$ and all extremals are of the form $s_c(x) = c\left(x - \frac{1}{2}\right)$ for any constant c.

Remark. While (2) is simple in form it is much harder to handle than (1). As in the previous case the main difficulty is caused by the absolute value signs on the left side, but the technique we used to prove (1) no longer seems applicable since it is hard to construct a piecewise monotone function s with the properties of Y while preserving the condition $\int_0^1 s\,dx = 0$.

However, one can verify the conjecture if certain assumptions are made about the extremals. For instance we can suppose:

(i) The extremal s is a linear spline with one knot.

(ii) The extremal up to multiplication by constants is unique.

If (i) is granted we can show that $K = \frac{1}{4}$ by an extremely laborious calculation. Since the argument based on (ii) is fairly short we present it here.

Lemma. Assumption $(ii) \Rightarrow K = \frac{1}{4}$ and $s(t) = t - \frac{1}{2}$ is an extremal.

Proof. Suppose there is an extremal s of (2). Because $\int_0^1 s = 0$, s has at least one zero $c \in (0, 1)$.

Case (1) If $c = \frac{1}{2}$, we know from a standard "half interval" Opial inequality [4, Theorem $2'$, p. 114] that

$$\int_0^{\frac{1}{2}} |ss'|\, dx \leq \frac{1}{2} \cdot \frac{1}{2} \int_0^{\frac{1}{2}} s'^2\, dx \tag{3}$$

$$\int_{\frac{1}{2}}^1 |ss'|\, dx \leq \frac{1}{2} \cdot \frac{1}{2} \int_{\frac{1}{2}}^1 s'^2\, dx. \tag{4}$$

From which it follows that s satisfies (2) with $K \leq \frac{1}{4}$.

Case (2) If $c \neq \frac{1}{2}$. Consider $\tilde{s}(t) = s(1-t)$. $\tilde{s}$ is also an extremal of (2). By hypothesis $\tilde{s}(t) = ks(t)$. Taking $t = \frac{1}{2}$ shows that $k = 1$. Hence if $t = \frac{1}{2} \pm u$, $1 - t = \frac{1}{2} \mp u$ and s is symmetric with respect to $t = \frac{1}{2}$. With no loss of generality we can assume that $c \in \left(0, \frac{1}{2}\right)$; there is then another zero $c' \in \left(\frac{1}{2}, 1\right)$. Again using [4, Theorem $2'$] yields that

$$\int_0^c |ss'|\, dx \leq \frac{c}{2} \int_0^{\frac{1}{2}} s'^2\, dx$$

$$\int_c^{\frac{1}{2}} |ss'|\, dx \leq \frac{\frac{1}{2} - c}{2} \int_0^{\frac{1}{2}} s'^2\, dx$$

which implies (3). The argument for (4) is similar using c'.

Thus in either Case (1) or (2) $K \leq \frac{1}{4}$. Since $s(t) = t - \frac{1}{2}$ gives equality in (2) and $\int_0^1 s = 0$, $K = \frac{1}{4}$. □

However, neither (i) nor (ii) is evident (although a calculus of variations argument will show that s is at least piecewise linear).

Another route to the solution of the problem may be to use a technique devised by Boyd [2] to find best constants in general Opial-like inequalities. Boyd considers the operator $K : L^2(0,1) \to L^2(0,1)$ defined by

$$Kf(x) = \int_0^1 k(x,t) f(t) \sigma(t)\, dt,$$

where $k(x,t)$ is nonnegative and measurable on $(0,1) \times (0,1)$, and σ is a positive a.e. measurable function. It is then shown [2, Theorem 1] that the best constant C of the inequality

$$\int_0^1 |K(f(x)|\, |f(x)| \sigma(x)\, dx \leq C \int_0^1 |f(x)|^2 \sigma(x)\, dx \tag{5}$$

is an eigenvalue of $(K + K^*)/2$. Boyd uses this method to find the best constants of a family of higher order generalizations of (1). However the calculations needed to put the inequality in the format (5) and to solve the eigenvalue problem are challenging.

REFERENCES

[1] R.P. AGARWAL AND P.Y.H. PANG, *Opial Inequalities with Applications in Differential and Difference Equations*, Kluwer Academic Publishers, Dordrecht, 1995.

[2] D.W. BOYD, Best constants in inequalities related to Opial's inequality, *J. Math. Anal. Appl.*, **24** (1969), 378–387.

[3] R.C. BROWN, A.M. FINK AND D.B. HINTON, Some Opial, Lyapunov, and De la Valée Poussin inequalities with nonhomogeneous boundary conditions, *J. Inequal. Appl.*, **5** (2000), 11–37.

[4] D. S. MITRINOVIĆ, J.E. PEČARIĆ AND A.M. FINK, *Inequalities Involving Functions and their Integrals and Derivatives*, Kluwer, Dordrecht, 1991.

[5] C. OLECH, A simple proof of a certain result of Z. Opial, *Ann. Polon. Math.*, **8** (1960) 61–63.

[6] Z. OPIAL, Sur une inégalité, *Ann. Polon. Math.*, **8** (1960), 29–32.

Richard Brown.

3.6. **Remark.**

Let $I \subset \mathbb{R}$ be an interval and $f, g : I \to \mathbb{R}$ be given functions, $f \le g$. It is well known that if f is concave and g is convex (or conversely), then there exists an affine function $h : I \to \mathbb{R}$ such that $f \le h \le g$ on I. Of course these conditions are sufficient but not necessary for the existence of such a function. A full characterization of functions which can be separated by an affine one gives the following theorem [5] (cf. also [2], [4] for further generalizations).

Theorem. Let $f, g : I \to \mathbb{R}$. There exists an affine function $h : I \to \mathbb{R}$ such $f \le h \le g$ if and only if

$$f(tx + (1-t)y) \le tg(x) + (1-t)g(y)$$

and

$$g(tx + (1-t)y) \ge tf(x) + (1-t)f(y)$$

for all $x, y \in I$ and $t \in [0, 1]$.

The first of the above inequalities is equivalent to the separability of f and g by a convex function (cf. [1]). From this result one can obtain (taking $g = f + \varepsilon$) the classical (one dimensional) Hyers-Ulam stability theorem [3] stating that if $f : I \to \mathbb{R}$ is ε-convex, i.e., it satisfies the condition

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y) + \varepsilon, \qquad x, y \in I, t \in [0, 1],$$

then there exists a convex function $h : I \to \mathbb{R}$ such that $f \le h \le f + \varepsilon$.

REFERENCES

[1] K. BARON, J. MATKOWSKI AND K. NIKODEM, A sandwich with convexity, *Math. Pannonica*, **5** (1994), 139–144.

[2] E. BEHRENDS AND K. NIKODEM, A selection theorem of Helly type and its applications, *Studia Math.*, **116** (1995), 43–48.

[3] D. H. HYERS AND S. M. ULAM, Approximately convex functions, *Proc. Amer. Math. Soc.*, **3** (1952), 821–828.

[4] K. NIKODEM, Zs. PÁLES AND Sz. WĄSOWICZ, Abstract separation theorems of Rodé type and their applications, *Ann. Polon. Math.*, **72** (1999), 207–217.

[5] K. NIKODEM AND Sz. WĄSOWICZ, A sandwich theorem and Hyers-Ulam stability of affine functions, *Aequationes Math.*, **49** (1995), 160–164.

Kazimierz Nikodem.

3.7. Problems.

Concerning Hardy type inequalities, we have posed the following problems.

Problem 1. Let $p > 1,\ 0 < \lambda < 1,\ 0 < b \leq \infty$ and $g \in C_0^\infty[0,b]$. Find necessary and sufficient conditions on the weights $u = u(x),\ 0 \leq x \leq b$, and $v = v(x,y),\ 0 \leq x, y \leq b$, so that

$$\left(\int_0^b |g(x)|^p\, u(x)dx\right)^{\frac{1}{p}} \leq K \left(\int_0^b \int_0^b \frac{|g(x)-g(y)|^p}{|x-y|^{1+\lambda p}} v(x,y)dxdy\right)^{\frac{1}{p}} \tag{1}$$

holds for some finite $K > 0$ and $\lambda \neq \frac{1}{p}$.

Remark 1. The (lower fractional order Hardy) inequality (1) holds, e.g., if $u(x) = x^{-p\lambda}$ and $v(x,y) \equiv 1$ except for $\lambda = \frac{1}{p}$, where a counterexample can be found.

Problem 2. Let $p > 1,\ 0 < \lambda < 1,\ 0 < b \leq \infty$ and $g \in AC[0,b]$. Find necessary and sufficient conditions on the weights $v = v(x),\ 0 \leq x \leq b$, and $u = u(x,y),\ 0 \leq x, y \leq b$, so that

$$\left(\int_0^b \int_0^b \frac{|g(x)-g(y)|^p}{|x-y|^{1+\lambda p}} u(x,y)dxdy\right)^{\frac{1}{p}} \leq K \left(\int_0^b |g'(x)|^p\, v(x)dx\right)^{\frac{1}{p}} \tag{2}$$

holds for some finite $K > 0$.

Remark 2. The (upper fractional order Hardy) inequality (2) holds e.g. if $u(x,y) = 1,\ v(x) = x^{(1-\lambda)p}$ and $K = 2^{\frac{1}{p}}\lambda^{-1}(p(1-\lambda))^{-\frac{1}{p}}$.

Problem 3 (A). Let $g \in L^2(0,\infty)$. Then

$$\|g\|_{L^2} = \|g - Hg\|_{L^2} = \|g - Sg\|_{L^2},$$

where

$$Hg(x) := \frac{1}{x}\int_0^x g(y)dy, \qquad Sg(x) = \int_x^\infty \frac{g(y)}{y}dy.$$

Question 1. Describe all the (averaging) operators A such that

$$\|g\|_{L^2} = \|g - Ag\|_{L^2}.$$

Problem 3 (B). Let $g \in L^p([0,\infty], x^{-\alpha p-1}) =: L^p(x^{-\alpha p -1})$ with $p \geq 1$ and $\alpha > -1$, $\alpha \neq 0$. Then

$$\|g\|_{L^p(x^{-\alpha p-1})} \approx \|g - Hg\|_{L^p(x^{-\alpha p-1})} \tag{3}$$

where

$$Hg(x) := \frac{1}{x}\int_0^x g(y)dy.$$

Question 2. Describe all the (averaging) operators A such that (3) holds with H replaced by A.

Lars-Erik Persson.

3.8. **Problem.**

If numbers $y_1, \dots, y_n$ (and $y_0 = 0$) are given positive numbers so that $\Delta y_k \geq 0$ ($k = 0, \dots, (n-1)$), $\Delta y_k = y_{k+1} - y_k$ and $\Delta^2 y_k \geq 0$ then there is a continuous convex function $\Delta^2 f \geq 0$ such that $f(i) = y_i$, $(i = 0, \dots, k)$. The piecewise linear function that interpolates the points (i, y_i) will do.

Now if I add the conditions that $\Delta^3 y_k \geq 0$ and ask for a continuous function f which interpolates the points (i, y_i) and $\Delta^3 f \geq 0$, then in general this cannot be done. So the problem is to find a finite set of conditions on the $\{y_i\}$ ensure the existence of a function f with the required properties.

A.M. Fink.

3.9. **Problems.**

Let $m \geq 2$ an integer and

$$\mu_m := \min_{|z|=1, z\in\mathbb{C}} \left| \sum_{k=1}^{m} k z^{m-k} \right|.$$

Problem 1. Find μ_m (as a function of m).

Problem 2. Prove or disprove that for odd m

$$\mu_m \geq \frac{m}{2} \sec \frac{\pi}{2m+2}. \tag{3.1}$$

Introducing $z = e^{it}$ we have

$$\left| \sum_{k=1}^{m} k z^{m-k} \right|^2 = \left[\frac{m}{2} + \frac{1}{2} \left(\frac{\sin \frac{mt}{2}}{\sin \frac{t}{2}} \right)^2 \right]^2 + \left[\frac{m \sin t - \sin mt}{4 \sin^2 \frac{t}{2}} \right]^2 \tag{3.2}$$

which shows that $\mu_m \geq \frac{m}{2}$, moreover $\mu_m = \frac{m}{2}$ if m is even as in this case the right hand side of (3.2) equals $\left(\frac{m}{2}\right)^2$ for $t = \pi$.

Problems 1 and 2 are related to the location of zeros of self-inversive polynomials through the following results.

Theorem 1. *(P. Lakatos* [1], [2]*) All zeros of reciprocal polynomial $P_m(z) = \sum_{k=1}^{m} A_k z^k$ with real coefficient $A_k \in \mathbb{R}$ (and $A_m \neq 0, \quad A_k = A_{m-k}$ for all $k = 1, \dots, n$) are on the unit circle, provided that*

$$|A_m| \geq \sum_{k=1}^{m-1} |A_k - A_m|. \tag{3.3}$$

Moreover, the zeros e^{iu_j} of P_m can be arranged such that

$$\left| e^{iu_j} - \varepsilon_j \right| \leq \frac{\pi}{m+1} \qquad (j = 1, \dots, m) \tag{3.4}$$

where $\varepsilon_j = e^{i\frac{2\pi j}{m+1}} \qquad (j = 1, \dots, m)$ are the $(m+1)$st roots of unity, except the root 1.

Theorem 2. *(A. Schinzel* [4]*.) All zeros of the self-inversive polynomial $P_m(z) = \sum_{k=1}^{m} A_k z^k$ (where $A_k \in \mathbb{C}, \quad A_m \neq 0,\ \varepsilon \bar{A}_k = A_{m-k}$ for all $k = 0, \dots, m$ with fixed $\varepsilon \in \mathbb{C},\ |\varepsilon| = 1$) are on the unit circle, provided that*

$$|A_m| \geq \inf_{c\in\mathbb{C}} \sum_{k=0}^{m} |cA_k - A_m|. \tag{3.5}$$

The first theorem has been generalized by proving that its statements remain valid if m is odd and (3.3) is replaced by

$$|A_m| \geq \cos^2 \frac{\pi}{2(m+1)} \sum_{k=1}^{m} |A_k - A_m|$$

and similarly, the second theorem remains valid if m is odd and (3.5) is replaced by

$$|A_m| \geq \frac{m}{2\mu_m} \sum_{k=0}^{m} |cA_k - A_m| .$$

(see Lakatos-Losonczi [3]).

REFERENCES

[1] P. LAKATOS, On polynomials having zeros on the unit circle, *C. R. Math. Acad. Soc. R. Canada*, **24** (2002), 91–96.

[2] P. LAKATOS, On zeros of reciprocal polynomials, *Publ. Math. Debrecen*, **61** (2002), 645–661.

[3] P. LAKATOS AND L. LOSONCZI, On zeros of reciprocal polynomials of odd degree, manuscript.

[4] A. SCHINZEL, Self-inversive polynomials with all zeros on the unit circle, manuscript.

László Losonczi.

3.10. **Remark and Problem.**

Let M and N be strict means in the usual sense, i.e.,

$$\min(x,y) < \left\{ \begin{array}{c} M(x,y) \\ N(x,y) \end{array} \right\} < \max(x,y)$$

if $x \neq y$ and $M(x,x) = N(x,x) = x$. Then the two sequences

$$x_1 = x,\ y_1 = y,\ x_{n+1} = M(x_n, y_n),\ y_{n+1} = N(x_n, y_n), \qquad (n = 1, 2, \dots)$$

converge to the same limit

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n := G_{M,N}(x,y).$$

Substituting x_n, y_n into

$$f(M(x,y)) + f(N(x,y)) \leq f(x) + f(y), \tag{1}$$

iterating the inequality, and tending to ∞ with n, we get, if f is continuous,

$$2f(G_{M,N}(x,y)) \leq f(x) + f(y). \tag{2}$$

Problem. Under what conditions on M and N does (2) follow from (1) without assuming continuity of f?

In what follows, we solve in two particular cases the corresponding *equations*

$$f(M(x,y)) + f(N(x,y)) = f(x) + f(y) \tag{1=}$$

$$2f(G_{M,N}(x,y)) = f(x) + f(y). \tag{2=}$$

1. $M(x,y) = \dfrac{x+y}{2}$, $N(x,y) = \dfrac{2xy}{x+y}$. Here $G_{M,N}(x,y) = \sqrt{xy}$. Hence the continuous solution of (2=) and thus also of (1=) is given by $f(x) = a\log x + b$. Without assuming any regularity, Bruce Ebanks recently proved (to appear in Publ. Math.) that the two equations have the same general solution $f(x) = \ell(x) + b$, where ℓ is an arbitrary solution of $\ell(xy) = \ell(x) + \ell(y)$.

2. $M(x,y) = \dfrac{x+y}{2}$, $N(x,y) = \sqrt{xy}$. In this case $G_{M,N}$ is Gauss's medium arithmetico-geometricum. It follows from a result of Gy. Maksa (*Publ. Math. Debrecen*, **24** (1977), 25–29), again without any regularity assumption, that every solution of (1=), that is, of

$$f\left(\frac{x+y}{2}\right) + f(\sqrt{xy}) = f(x) + f(y)$$

is constant.

Remark. The above two results would not be surprising if f were supposed (continuous and) strictly monotonic. Then (2=) would become

$$G_{M,N}(x,y) = f^{-1}\left(\frac{f(x)+f(y)}{2}\right),$$

making $G_{M,N}$ a quasi-arithmetic mean. Now, $G_{M,N}(x,y) = \sqrt{xy}$ is a quasi-arithmetic mean with $f(x) = a\log x + b$ $(a \neq 0)$ while the medium arithmetico-geometricum is not quasi-arithmetic. What is surprising is that the statements **1** and **2** hold without any regularity assumption.

János Aczél and Zsolt Páles.

3.11. **Remark.**

Lars-Erik Persson has asked for an elementary proof of the identities

$$\|f - A_i f\|_{L^2(0,\infty)} = \|f\|_{L^2(0,\infty)} \qquad \left(f \in L^2(0,\infty);\ i = 1,2\right),$$

with A_1 and A_2 denoting the averaging operators

$$(A_1 f)(x) := \frac{1}{x}\int_0^x f(t)dt, \qquad (A_2 f)(x) := \int_x^\infty \frac{1}{t} f(t)dt.$$

Indeed, for $f \in L^2(0,\infty)$,

$$\begin{aligned}
\|f\|^2_{L^2(0,\infty)} - \|f - A_1 f\|^2_{L^2(0,\infty)} &= 2\operatorname{Re}\langle A_1 f, f\rangle_{L^2(0,\infty)} - \|A_1 f\|^2_{L^2(0,\infty)} \\
&= \int_0^\infty \frac{1}{x}\cdot 2\operatorname{Re}\left[\int_0^x f(t)dt \cdot \overline{f(x)}\right] dx - \|A_1 f\|^2_{L^2(0,\infty)} \\
&= \int_0^\infty \frac{1}{x}\frac{d}{dx}\left|\int_0^x f(t)dt\right|^2 dx - \int_0^\infty \frac{1}{x^2}\left|\int_0^x f(t)dt\right|^2 dx \\
&= \left[\frac{1}{x}\left|\int_0^x f(t)dt\right|^2\right]_0^\infty
\end{aligned}$$

by partial integration. So it suffices to show that

$$\lim_{x\to 0}\frac{1}{x}\left|\int_0^x f(t)dt\right|^2 = \lim_{x\to\infty}\frac{1}{x}\left|\int_0^x f(t)dt\right|^2 = 0.$$

The first limit being 0 is immediate by the Cauchy-Schwarz inequality:

$$\frac{1}{x}\left|\int_0^x f(t)dt\right|^2 \leq \int_0^x |f(t)|^2\,dt \longrightarrow 0 \text{ as } x \to 0.$$

For the second, we observe that, for any $0 < y < x < \infty$,

$$\begin{aligned} 0 &\leq \frac{1}{x}\left|\int_0^x f(t)dt\right|^2 \\ &\leq \frac{1}{x}\left[\int_0^y |f(t)|\,dt + \int_y^x |f(t)|\,dt\right]^2 \\ &\leq \frac{2}{x}\left[\int_0^y |f(t)|\,dt\right]^2 + \frac{2}{x}\left[\int_y^x |f(t)|\,dt\right]^2 \\ &\leq \frac{2}{x}\left[\int_0^y |f(t)|\,dt\right]^2 + \frac{2}{x}(x-y)\int_y^x |f(t)|^2\,dt \\ &\leq \frac{2}{x}\left[\int_0^y |f(t)|\,dt\right]^2 + 2\int_y^\infty |f(t)|^2\,dt. \end{aligned}$$

For given $\varepsilon > 0$, choose now y such that second term is less than $\varepsilon/2$, and then $x_0 > y$ such that the first term is less than $\varepsilon/2$ (for $x \geq x_0$).

For the averaging operator A_2, we obtain similarly

$$\begin{aligned} \|f\|^2_{L^2(0,\infty)} - \|f - A_2 f\|^2_{L^2(0,\infty)} &= \int_0^\infty 2\,\mathrm{Re}\left[\int_x^\infty \frac{1}{t} f(t)dt \cdot \overline{f(x)}\right] dx - \|A_2 f\|^2_{L^2(0,\infty)} \\ &= -\int_0^\infty x\frac{d}{dx}\left|\int_x^\infty \frac{1}{t}f(t)dt\right|^2 dx - \int_0^\infty \left|\int_x^\infty \frac{1}{t}f(t)dt\right|^2 dx \\ &= -\left[x\left|\int_x^\infty \frac{1}{t}f(t)dt\right|^2\right]_0^\infty, \end{aligned}$$

and

$$\begin{aligned} x\left|\int_x^\infty \frac{1}{t}f(t)dt\right|^2 &\leq x\int_x^\infty \frac{1}{t^2}dt \int_x^\infty |f(t)|^2\,dt \\ &= \int_x^\infty |f(t)|^2\,dt \longrightarrow 0 \text{ as } x \to \infty. \end{aligned}$$

To prove that the above boundary term vanishes also at 0, let $0 < x < y < \infty$. Then,

$$\begin{aligned} x\left|\int_x^\infty \frac{1}{t}f(t)dt\right|^2 &\leq 2x\left[\int_x^y \frac{1}{t}|f(t)|\,dt\right]^2 + 2x\left[\int_y^\infty \frac{1}{t}|f(t)|\,dt\right]^2 \\ &\leq 2x\int_x^y \frac{1}{t^2}dt\int_x^y |f(t)|^2\,dt + 2x\left[\int_y^\infty \frac{1}{t}|f(t)|\,dt\right]^2 \\ &\leq 2\int_0^y |f(t)|^2\,dt + 2x\left[\int_y^\infty \frac{1}{t}|f(t)|\,dt\right]^2 \end{aligned}$$

which is less than a given $\varepsilon > 0$ (for y sufficiently small and x sufficiently small depending on y), similarly to the arguments for A_1.

Michael Plum.

3.12. **Remark.**

We describe solutions to the functional inequality

$$\|f(x+y)-f(x)-f(y)\| \leq \varepsilon\left(\|x\|+\|y\|\right)$$

for $f:\mathbb{R}^n \to X$.

Jacek Tabor.

3.13. **Problem.**

Let $-\infty < \alpha, \beta < \infty$ and consider for $f \geq 0$ the (Gini) means

$$G_{\alpha,\beta}[f,x] = \begin{cases} \left(\dfrac{\int_0^x f^\alpha(t)dt}{\int_0^x f^\beta(t)dt}\right)^{\frac{1}{\alpha-\beta}}, & \alpha \neq \beta \\ \exp\left(\dfrac{\int_0^x f^\alpha(t)\log f(t)dt}{\int_0^x f^\alpha(t)dt}\right), & \alpha = \beta. \end{cases}$$

Let also $\{p,q\} \in \mathbb{R}_+^2$ (or to some suitable subset of $\mathbb{R}_+^2$),

a) Find necessary and sufficient conditions on the weights $u(x)$ and $v(x)$ so that, for $0 < b \leq \infty$,

$$\left(\int_0^b \left(G_{\alpha,\beta}[f,x]\right)^q u(x)dx\right)^{\frac{1}{q}} \leq C\left(\int_0^b f^p(x)v(x)dx\right)^{\frac{1}{p}} \tag{1}$$

for some finite $C > 0$. Find also "good" estimates of the least constant C in (1) [i.e. good control of the corresponding operator norm].

b) The same question when $G_{\alpha,\beta}[f,x]$ is replaced by other interesting means, e.g., those presented on this conference.

Remark 1. For the case $\alpha = 1,\ \beta = 0,\ p > 1,\ q > 0$ (1) is just a modern form of Hardy's inequality so we have a complete solution of our problem. Also, according to results e.g. presented in this conference we have the similar precise information for the geometric mean case $\alpha = \beta = 0,\ p, q > 0$. Moreover, for the power mean case $\alpha > 0,\ \beta = 0$ we also get satisfactory results by just making an obvious substitution in the arithmetic mean (=the Hardy) case.

Remark 2. The scale of means $G_{\alpha,\beta}$ has the interesting property that it is increasing in both α and β. This means that we can get sufficient conditions when (1) holds by just using the information pointed out in Remark 1.

Alois Kufner, Zsolt Páles, and Lars-Erik Persson.

3.14. **Remark.**

Saborou Saitoh has asked for a simple proof of the inequality

$$\inf_{\|g\|\leq R} \int_{-\infty}^{\infty} |u_f(x,t)-u_g(x,t)|^2\,dx \leq (\|f\|-R)^2$$

for all $f \in L^2(\mathbb{R})$, $\|f\| \geq R$, and $t \geq 0$, where

$$u_f(x,t) := \frac{1}{\sqrt{4\pi t}}\int_{-\infty}^{\infty} f(y)\exp\left(-\frac{|x-y|^2}{4t}\right)dy.$$

Indeed, u_f solves the Cauchy problem for the heat equation,

$$\frac{\partial u_f}{\partial t} = \Delta u_f \qquad (x \in \mathbb{R},\ t > 0), \quad u_f(\cdot, 0) = f \text{ a.e. on } \mathbb{R}.$$

Then, for f and g in $L^2(\mathbb{R})$ and all $t > 0$,

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\|u_f(\cdot,t) - u_g(\cdot,t)\|^2 &= \left\langle \frac{\partial u_f}{\partial t}(\cdot,t) - \frac{\partial u_g}{\partial t}(\cdot,t), u_f(\cdot,t) - u_g(\cdot,t) \right\rangle_{L^2(\mathbb{R})} \\ &= \langle \Delta u_f(\cdot,t) - \Delta u_g(\cdot,t), u_f(\cdot,t) - u_g(\cdot,t)\rangle_{L^2(\mathbb{R})} \\ &= -\|\nabla u_f(\cdot,t) - \nabla u_g(\cdot,t)\|^2 \leq 0 \end{aligned}$$

by partial integration since $u(\cdot,t)$ and $\nabla u(\cdot,t)$ decay exponentially at $x = \pm\infty$. Thus,

$$\|u_f(\cdot,t) - u_g(\cdot,t)\|^2 \leq \|u_f(\cdot,0) - u_g(\cdot,0)\|^2 = \|f - g\|^2.$$

For $\|f\| \geq R$ and $g := \frac{R}{\|f\|}f$, we therefore have

$$\|u_f(\cdot,t) - u_g(\cdot,t)\|^2 \leq \left(1 - \frac{R}{\|f\|}\right)^2 \|f\|^2 = (\|f\| - R)^2$$

and $\|g\| = R$, which establishes the result.

Michael Plum.

4. ADDENDA

4.1. **Addenda zu *Gewöhnliche Differentialgleichungen mit quasimonoton wachsenden rechten Seiten in geordneten Banachräumen.*** von Alice Chaljub-Simon, Roland Lemmert, Sabina Schmidt und Peter Volkmann, General Inequalities 6, International Series of Numerical Mathematics 103, Birkhäuser, Basel, 1992, pp. 307-320.

1. In Lemma 2 auf S. 311 soll II) wie folgt lauten:

$$\text{II) Aus } x, y \in E,\ x \leq y,\ x_\alpha = y_\alpha \text{ folgt } f_\alpha(x) \leq f_\alpha(y).$$

(In dieser Form wird II) später benutzt; der Beweis ist ähnlich dem Beweise der ursprünglichen Form von II).)

2. Auf S. 317, 14.-16. Z. v.o. sind die beiden Sätze “In Wirklichkeit ... verfeinert werden.” zu ersetzen durch: Für normale Kegel K brauchte nur $f : [0,T] \times U \to E$ mit einer Umgebung U von a vorausgesetzt zu werden, und dann konnte die Existenz einer lokalen Lösung von (12) gezeigt werden. (Vgl. die Folgerung auf S. 388 von [6]; die Normalität von K ist dort den Voraussetzungen hinzuzufügen!) Entsprechende lokale Versionen können auch für den hier gegebenen Satz 2 bewiesen werden.

3. Auf S. 317, 8.-1. Z. v.u. (durch den Druck entstellt) soll es heißen:

$$C^+(M) = \{x \mid x \in C(M),\ x = (x_\alpha)_{\alpha\in M} \text{ mit } x_\alpha \geq 0\ (\alpha \in M)\}$$

der natürliche Ordnungskegel in $C(M)$.

SATZ 3. *Ist M ein metrischer Raum, dessen Metrik nicht diskret ist (d.h. in M gibt es mindestens einen Häufungspunkt), und wird $E = C(M)$ geordnet durch $K = C^+(M)$, so gibt es eine stetige, beschränkte, monoton wachsende Funktion $f : E \to E$ und ein $a \in E$ derart, daß das Anfangswertproblem*

$$u(0) = a, \quad u'(t) = f(u(t)) \qquad (0 \leq t \leq T) \tag{37}$$

für jedes $T > 0$ unlösbar ist.

Peter Volkmann.

4.2. **Addendum to *Weak persistence in Lotka-Volterra populations*.** by Raymond M. Redheffer and Peter Volkmann, General Inequalities 7, International Series of Numerical Mathematics 123, Birkhäuser, Basel, 1997, pp. 369-373.

The remark on page 371 pertaining to $y_1 - y_2$ overlooks the fact that $f(t)$ and $g(t)$ depend on y. The statement of Example 1 remains unchanged but the assertion that $\lim m(t) = 1$ in Example 2 should be replaced by $1/5 \leq \liminf m(t) \leq \limsup m(t) \leq 3$. This oversight was brought to our attention by Dr. Roland Uhl.

Peter Volkmann.

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