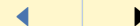
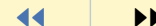


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FOURIER RESTRICTION ESTIMATES TO MIXED HOMOGENEOUS SURFACES

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Received: 17 September, 2008

Accepted: 13 February, 2009

Communicated by: L. Pick

2000 AMS Sub. Class.: Primary 42B10, 26D10.

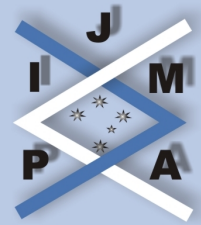
Key words: Restriction theorems, Fourier transform.

Abstract: Let a, b be real numbers such that $2 \leq a < b$, and let $\varphi : \mathbb{R}^2 \rightarrow \mathbb{R}$ a mixed homogeneous function. We consider polynomial functions φ and also functions of the type $\varphi(x_1, x_2) = A|x_1|^a + B|x_2|^b$. Let $\Sigma = \{(x, \varphi(x)) : x \in B\}$ with the Lebesgue induced measure. For $f \in S(\mathbb{R}^3)$ and $x \in B$, let $(\mathcal{R}f)(x, \varphi(x)) = \widehat{f}(x, \varphi(x))$, where \widehat{f} denotes the usual Fourier transform. For a large class of functions φ and for $1 \leq p < \frac{4}{3}$ we characterize, up to endpoints, the pairs (p, q) such that \mathcal{R} is a bounded operator from $L^p(\mathbb{R}^3)$ on $L^q(\Sigma)$. We also give some sharp $L^p \rightarrow L^2$ estimates.

Acknowledgements: Research partially supported by Secyt-UNC, Agencia Nacional de Promoción Científica y Tecnológica.
The authors wish to thank Professor Fulvio Ricci for fruitful conversations about this subject.

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issn: 1443-5756



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1. Introduction

Let a, b be real numbers such that $2 \leq a < b$, let $\varphi : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a mixed homogeneous function of degree one with respect to the non isotropic dilations $r \cdot (x_1, x_2) = \left(r^{\frac{1}{a}} x_1, r^{\frac{1}{b}} x_2 \right)$, i.e.

$$(1.1) \quad \varphi \left(r^{\frac{1}{a}} x_1, r^{\frac{1}{b}} x_2 \right) = r \varphi (x_1, x_2), \quad r > 0.$$

We also suppose φ to be smooth enough. We denote by B the closed unit ball of \mathbb{R}^2 , by

$$\Sigma = \{(x, \varphi(x)) : x \in B\}$$

and by σ the induced Lebesgue measure. For $f \in S(\mathbb{R}^3)$, let $\mathcal{R}f : \Sigma \rightarrow \mathbb{C}$ be defined by

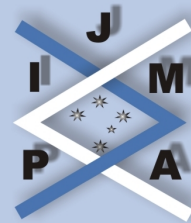
$$(1.2) \quad (\mathcal{R}f)(x, \varphi(x)) = \widehat{f}(x, \varphi(x)), \quad x \in B,$$

where \widehat{f} denotes the usual Fourier transform of f . We denote by E the type set associated to \mathcal{R} , given by

$$E = \left\{ \left(\frac{1}{p}, \frac{1}{q} \right) \in [0, 1] \times [0, 1] : \|\mathcal{R}\|_{L^p(\mathbb{R}^3), L^q(\Sigma)} < \infty \right\}.$$

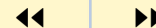
Our aim in this paper is to obtain as much information as possible about the set E , for certain surfaces Σ of the type above described.

In the general n -dimensional case, the $L^p(\mathbb{R}^{n+1}) - L^q(\Sigma)$ boundedness properties of the restriction operator \mathcal{R} have been studied by different authors. A very interesting survey about recent progress in this research area can be found in [11]. The $L^p(\mathbb{R}^{n+1}) - L^2(\Sigma)$ restriction theorems for the sphere were proved by E. Stein in



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1967, for $\frac{3n+4}{4n+4} < \frac{1}{p} \leq 1$; for $\frac{n+4}{2n+4} < \frac{1}{p} \leq 1$ by P. Tomas in [12] and then in the same year by Stein for $\frac{n+4}{2n+4} \leq \frac{1}{p} \leq 1$. The last argument has been used in several related contexts by R. Strichartz in [9] and by A. Greenleaf in [6]. This method provides a general tool to obtain, from suitable estimates for $\widehat{\sigma}$, $L^p(\mathbb{R}^{n+1}) - L^2(\Sigma)$ estimates for \mathcal{R} . Moreover, a general theorem, due to Stein, holds for smooth enough hypersurfaces with never vanishing Gaussian curvature ([8], pp.386). There it is shown that in this case, $(\frac{1}{p}, \frac{1}{q}) \in E$ if $\frac{n+4}{2n+4} \leq \frac{1}{p} \leq 1$ and $-\frac{n+2}{n} \frac{1}{p} + \frac{n+2}{n} \leq \frac{1}{q} \leq 1$, also that this last relation is the best possible and that no restriction theorem of any kind can hold for $f \in L^p(\mathbb{R}^{n+1})$ when $\frac{1}{p} \leq \frac{n+2}{2n+2}$ ([8, pp.388]). The cases $\frac{n+2}{2n+2} < \frac{1}{p} < \frac{n+4}{2n+4}$ are not completely solved. The best results for surfaces with non vanishing curvature like the paraboloid and the sphere are due to T. Tao [10]. Restriction theorems for the Fourier transform to homogeneous polynomial surfaces in \mathbb{R}^3 are obtained in [4]. Also, in [1] the authors obtain sharp $L^p(\mathbb{R}^{n+l}) - L^2(\Sigma)$ estimates for certain homogeneous surfaces Σ of codimension l in \mathbb{R}^{n+l} .

In Section 2 we give some preliminary results.

In Section 3 we consider $\varphi(x_1, x_2) = A|x_1|^a + B|x_2|^b$, $A \neq 0, B \neq 0$. We describe completely, up to endpoints, the pairs $(\frac{1}{p}, \frac{1}{q}) \in E$ with $\frac{1}{p} > \frac{3}{4}$. A fundamental tool we use is Theorem 2.1 of [2].

In Section 4 we deal with polynomial functions φ . Under certain hypothesis about φ we can prove that if $\frac{3}{4} < \frac{1}{p} \leq 1$ and the pair $(\frac{1}{p}, \frac{1}{q})$ satisfies some sharp conditions, then $(\frac{1}{p}, \frac{1}{q}) \in E$. Finally we obtain some $L^{\frac{4}{3}} - L^q$ estimates and also some sharp $L^p - L^2$ estimates.



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2. Preliminaries

We take φ to be a mixed homogeneous and smooth enough function that satisfies (1.1). If V is a measurable set in \mathbb{R}^2 , we denote $\Sigma^V = \{(x, \varphi(x)) : x \in V\}$ and σ^V as the associated surface measure. Also, for $f \in S(\mathbb{R}^3)$, we define $\mathcal{R}^V f : \Sigma^V \rightarrow \mathbb{C}$ by

$$(\mathcal{R}^V f)(x, \varphi(x)) = \widehat{f}(x, \varphi(x)) \quad x \in V;$$

we note that $\mathcal{R}^B = \mathcal{R}$, $\sigma^B = \sigma$ and $\Sigma^B = \Sigma$.

For $x = (x_1, x_2)$ letting $\|x\| = |x_1|^a + |x_2|^b$, we define

$$A_0 = \left\{ x \in \mathbb{R}^2 : \frac{1}{2} \leq \|x\| \leq 1 \right\}$$

and for $j \in \mathbb{N}$,

$$A_j = 2^{-j} \cdot A_0.$$

Thus $B \subseteq \overline{\bigcup_{j \in \mathbb{N} \cup \{0\}} A_j}$. A standard homogeneity argument (see, e.g. [5]) gives, for $1 \leq p, q \leq \infty$,

$$(2.1) \quad \|\mathcal{R}^{A_j}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{A_j})} = 2^{-j \frac{a+b}{ab} \left(\frac{1}{q} - \frac{a+b+ab}{a+b} \frac{1}{p} + \frac{a+b+ab}{a+b} \right)} \|\mathcal{R}^{A_0}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{A_0})}.$$

From this we obtain the following remarks.

Remark 1. If $\left(\frac{1}{p}, \frac{1}{q}\right) \in E$ then $\frac{1}{q} \geq -\frac{a+b+ab}{a+b} \frac{1}{p} + \frac{a+b+ab}{a+b}$.

Remark 2. If $-\frac{a+b+ab}{a+b} \frac{1}{p} + \frac{a+b+ab}{a+b} < \frac{1}{q} \leq 1$ and

$$(2.2) \quad \|\mathcal{R}^{A_0}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{A_0})} < \infty,$$

then $\left(\frac{1}{p}, \frac{1}{q}\right) \in E$.



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We will use a theorem due to Strichartz (see [9]), whose proof relies on the Stein complex interpolation theorem, which gives $L^p(\mathbb{R}^3) - L^2(\Sigma^V)$ estimates for the operator \mathcal{R}^V depending on the behavior at infinity of $\widehat{\sigma^V}$. In [4] we obtained information about the size of the constants. There we found the following:

Remark 3. If V is a measurable set in \mathbb{R}^2 of positive measure and if

$$\left| \widehat{\sigma^V}(\xi) \right| \leq A(1 + |\xi_3|)^{-\tau}$$

for some $\tau > 0$ and for all $\xi = (\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3$, then there exists a positive constant c_τ such that

$$\left\| \mathcal{R}^V \right\|_{L^p(\mathbb{R}^3), L^2(\Sigma^V)} \leq c_\tau A^{\frac{1}{2(1+\tau)}}$$

for $p = \frac{2+2\tau}{2+\tau}$.

In [2] the authors obtain a result (Theorem 2.1, p.155) from which they also obtain the following consequence

Remark 4 ([2, Corollary 2.2]). Let I, J be two real intervals, and let

$$M = \{(x_1, x_2, \psi(x_1, x_2)) : (x_1, x_2) \in I \times J\},$$

where $\psi : I \times J \rightarrow \mathbb{R}$ is a smooth function such that either $\left| \frac{\partial^2 \psi}{\partial x_1^2}(x_1, x_2) \right| \geq c > 0$ or $\left| \frac{\partial^2 \psi}{\partial x_2^2}(x_1, x_2) \right| \geq c > 0$, uniformly on $I \times J$. If M has the Lebesgue surface measure, $\frac{1}{q} = 3 \left(1 - \frac{1}{p}\right)$ and $\frac{3}{4} < \frac{1}{p} \leq 1$ then there exists a positive constant c such that

$$(2.3) \quad \left\| \widehat{f}|_M \right\|_{L^q(M)} \leq c \|f\|_{L^p(\mathbb{R}^3)}$$

for $f \in S(\mathbb{R}^3)$.



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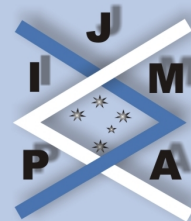
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Following the proof of Theorem 2.1 in [2] we can check that if in the last remark we take $J = [2^{-k}, 2^{-k+1}]$, $k \in \mathbb{N}$ in the case that $\left| \frac{\partial^2 \psi}{\partial x_1^2}(x_1, x_2) \right| \geq c > 0$ uniformly on $I \times J$ with c independent of k , or $I = [2^{-k}, 2^{-k+1}]$, $k \in \mathbb{N}$ in the other case, then we can replace (2.3) by

$$(2.4) \quad \left\| \widehat{f}|_M \right\|_{L^q(M)} \leq c' 2^{-k(\frac{1}{p} + \frac{1}{q} - 1)} \|f\|_{L^p(\mathbb{R}^3)}$$

with c' independent of k .



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3. The Cases $\varphi(x_1, x_2) = A|x_1|^a + B|x_2|^b$

In this cases we characterize, up to endpoints, the pairs $(\frac{1}{p}, \frac{1}{q}) \in E$ with $\frac{3}{4} < \frac{1}{p} \leq 1$. We also obtain some border segments. If either $A = 0$ or $B = 0$, φ becomes homogeneous and these cases are treated in [4]. For the remainder situation we obtain the following

Theorem 3.1. *Let $a, b, A, B \in \mathbb{R}$ with $2 \leq a \leq b$, $A \neq 0$, $B \neq 0$, let $\varphi(x_1, x_2) = A|x_1|^a + B|x_2|^b$ and let E be the type set associated to φ . If $\frac{3}{4} < \frac{1}{p} \leq 1$ and $-\frac{a+b+ab}{a+b} \frac{1}{p} + \frac{a+b+ab}{a+b} < \frac{1}{q} \leq 1$ then $(\frac{1}{p}, \frac{1}{q}) \in E$.*

Proof. Suppose $\frac{3}{4} < \frac{1}{p} \leq 1$ and $-\frac{a+b+ab}{a+b} \frac{1}{p} + \frac{a+b+ab}{a+b} < \frac{1}{q} \leq 1$. By Remark 2 it is enough to prove (2.2). Now, A_0 is contained in the union of the rectangles $Q = [-1, 1] \times [\frac{1}{2}, 1]$, $Q' = [\frac{1}{2}, 1] \times [-1, 1]$, and its symmetric with respect to the x_1 and x_2 axes. Now we will study $\|\mathcal{R}^Q\|_{L^p(\mathbb{R}^3), L^q(\Sigma^Q)}$. We decompose $Q = \bigcup_{k \in \mathbb{N}} Q_k$ with

$$Q_k = ([-2^{-k+1}, -2^{-k}] \cup [2^{-k}, 2^{-k+1}]) \times \left[\frac{1}{2}, 1\right].$$

Now, as in Theorem 1, (3.2), in [3] we have

$$\left| \widehat{\sigma^{Q_k}}(\xi) \right| \leq A 2^{k \frac{a-2}{2}} (1 + |\xi_3|)^{-1}$$

and then Remark 3 implies

$$(3.1) \quad \|\mathcal{R}^{Q_k}\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^2(\Sigma^{Q_k})} \leq c 2^{k \frac{a-2}{8}}.$$



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Also, since $\left| \frac{\partial^2 \varphi}{\partial x_2^2}(x_1, x_2) \right| \geq c > 0$ uniformly on Q_k , from (2.4) we obtain

$$\left\| \mathcal{R}^{Q_k} \right\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{Q_k})} \leq c' 2^{-k(\frac{1}{p} + \frac{1}{q} - 1)}$$

for $\frac{1}{q} = 3 \left(1 - \frac{1}{p}\right)$ and $\frac{3}{4} < \frac{1}{p} \leq 1$. Applying the Riesz interpolation theorem and then performing the sum on $k \in \mathbb{N}$ we obtain

$$\left\| \mathcal{R}^Q \right\|_{L^p(\mathbb{R}^3), L^q(\Sigma^Q)} < \infty,$$

for $\frac{2+3a}{2+a} \left(1 - \frac{1}{p}\right) < \frac{1}{q} \leq 1$ and $\frac{3}{4} < \frac{1}{p} \leq 1$. In a similar way we get that

$$\left\| \mathcal{R}^{Q'} \right\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{Q'})} < \infty,$$

for $\frac{2+3b}{2+b} \left(1 - \frac{1}{p}\right) < \frac{1}{q} \leq 1$ and $\frac{3}{4} < \frac{1}{p} \leq 1$. The study for the symmetric rectangles is analogous. Thus

$$\left\| \mathcal{R}^{A_0} \right\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{A_0})} < \infty$$

for $\frac{3}{4} < \frac{1}{p} \leq 1$ and $-\frac{a+b+ab}{a+b} \frac{1}{p} + \frac{a+b+ab}{a+b} < \frac{1}{q} \leq 1$ and the theorem follows. \square

Remark 5.

i) If $\frac{b+2}{8} < \frac{1}{q} \leq 1$ then $\left(\frac{3}{4}, \frac{1}{q}\right) \in E$.

ii) The point $\left(\frac{a+b+2ab}{2a+2b+2ab}, \frac{1}{2}\right) \in E$.



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From (3.1) and the Hölder inequality we obtain that

$$\|\mathcal{R}^{Q_k}\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^q(\Sigma^{Q_k})} \leq c2^k \left(\frac{a-2}{8} - \frac{2-q}{2q} \right)$$

for $\frac{1}{2} \leq \frac{1}{q} \leq 1$. Then if $\frac{a+2}{8} < \frac{1}{q} \leq 1$ we perform the sum over $k \in \mathbb{N}$ to get

$$\|\mathcal{R}^Q\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^q(\Sigma^Q)} < \infty,$$

for these q 's. Analogously, if $\frac{b+2}{8} < \frac{1}{q} \leq 1$ we get

$$\|\mathcal{R}^{Q'}\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^q(\Sigma^{Q'})} < \infty,$$

thus since $a \leq b$, if $\frac{b+2}{8} < \frac{1}{q} \leq 1$,

$$\|\mathcal{R}^{A_0}\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^q(\Sigma^{A_0})} < \infty,$$

and *i*) follows from Remark 2.

Assertion *ii*) follows from Remark 3, since from Lemma 3 in [3] we have that

$$|\widehat{\sigma}(\xi)| \leq c(1 + |\xi_3|)^{-\frac{1}{a} - \frac{1}{b}}.$$



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4. The Polynomial Cases

In this section we deal with mixed homogeneous polynomial functions φ satisfying (1.1). The following result is sharp (up to the endpoints) for $\frac{3}{4} < \frac{1}{p} \leq 1$, as a consequence of Remark 1.

Theorem 4.1. *Let φ be a mixed homogeneous polynomial function satisfying (1.1). Suppose that the gaussian curvature of Σ does not vanish identically and that at each point of $\Sigma^{B-\{0\}}$ with vanishing curvature, at least one principal curvature is different from zero. If $(a, b) \neq (2, 4)$, $\frac{3}{4} < \frac{1}{p} \leq 1$ and $-\frac{a+b+ab}{a+b} \frac{1}{p} + \frac{a+b+ab}{a+b} < \frac{1}{q} \leq 1$ then $(\frac{1}{p}, \frac{1}{q}) \in E$.*

Proof. We first study the operator \mathcal{R}^{A_0} . Let $(x_1^0, x_2^0) \in A_0$. If $Hess\varphi(x_1^0, x_2^0) \neq 0$ there exists a neighborhood U of (x_1^0, x_2^0) such that $Hess\varphi(x_1, x_2) \neq 0$ for $(x_1, x_2) \in U$. From the proposition in [8, pp. 386], it follows that

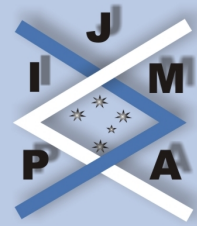
$$(4.1) \quad \|\mathcal{R}^U\|_{L^p(\mathbb{R}^3), L^q(\Sigma^U)} < \infty$$

for $\frac{1}{q} = 2 \left(1 - \frac{1}{p}\right)$ and $\frac{3}{4} \leq \frac{1}{p} \leq 1$. Suppose now that $Hess\varphi(x_1^0, x_2^0) = 0$ and that either $\frac{\partial^2 \varphi}{\partial x_1^2}(x_1^0, x_2^0) \neq 0$ or $\frac{\partial^2 \varphi}{\partial x_2^2}(x_1^0, x_2^0) \neq 0$. Then there exists a neighborhood $V = I \times J$ of (x_1^0, x_2^0) such that either $\left| \frac{\partial^2 \varphi}{\partial x_1^2}(x_1, x_2) \right| \geq c > 0$ or $\left| \frac{\partial^2 \varphi}{\partial x_2^2}(x_1, x_2) \right| \geq c > 0$ uniformly on V . So from Remark 4 we obtain that

$$(4.2) \quad \|\mathcal{R}^V\|_{L^p(\mathbb{R}^3), L^q(\Sigma^V)} < \infty$$

for $\frac{1}{q} = 3 \left(1 - \frac{1}{p}\right)$ and $\frac{3}{4} < \frac{1}{p} \leq 1$. From (4.1), (4.2) and Hölder's inequality, it follows that

$$(4.3) \quad \|\mathcal{R}^{A_0}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{A_0})} < \infty$$



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for $\frac{1}{q} \geq 3 \left(1 - \frac{1}{p}\right)$ and $\frac{3}{4} < \frac{1}{p} \leq 1$. So, if $\frac{a+b+ab}{a+b} \geq 3$, the theorem follows from Remark 2. The only cases left are $(a, b) = (3, 4)$, $(a, b) = (3, 5)$, $(a, b) = (4, 5)$ and $(a, b) = (2, b)$, $b > 2$. If $(a, b) = (3, 4)$ and φ has a monomial of the form $a_{i,j}x^i y^j$, with $a_{ij} \neq 0$, then $\frac{i}{3} + \frac{j}{4} = 1$ so $4i + 3j = 12$ and so either $(i, j) = (0, 4)$ or $(i, j) = (3, 0)$. So $\varphi(x_1, x_2) = a_{3,0}x_1^3 + a_{0,4}x_2^4$. The hypothesis about the derivatives of φ imply that $a_{3,0} \neq 0$ and $a_{0,4} \neq 0$ and the theorem follows using Theorem 3.1 in each quadrant. The cases $(a, b) = (3, 5)$, or $(a, b) = (4, 5)$ are completely analogous.

Now we deal with the cases $(a, b) = (2, b)$, $b > 2$. We note that

$$(4.4) \quad \varphi(x_1, x_2) = Ax_1^2 + Bx_1x_2^{\frac{b}{2}} + Cx_2^b$$

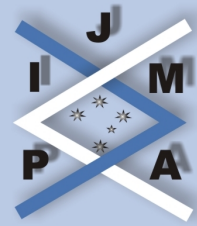
where $B = 0$ for b odd. The hypothesis about φ implies $A \neq 0$. For b odd, $\varphi(x_1, x_2) = Ax_1^2 + Cx_2^b$ and since $C \neq 0$ (on the contrary $Hess\varphi(x_1, x_2) \equiv 0$), the theorem follows using Theorem 3.1 as before. Now we consider b even and φ given by (4.4). If $B = 0$ the theorem follows as above, so we suppose $B \neq 0$.

$$(4.5) \quad Hess\varphi(x_1, x_2) = -\frac{x_2^{\frac{b}{2}-2}}{4} \left((B^2b^2 + 8ACb - 8ACb^2) x_2^{\frac{b}{2}} - 2(b-2)ABbx_1 \right).$$

So if $Hess\varphi(x_1^0, x_2^0) = 0$ then either $x_2^0 = 0$ or

$$(B^2b^2 + 8ACb - 8ACb^2) (x_2^0)^{\frac{b}{2}} - 2(b-2)ABbx_1^0 = 0.$$

In the first case we have $b > 4$. We take a neighborhood $W_1 = I \times [-2^{-k_0}, 2^{-k_0}] \subset A_0$, $k_0 \in \mathbb{N}$, of the point $(x_1^0, 0)$ such that $Hess\varphi$ vanishes, on W_1 , only along the x_1 axes. For $k \in \mathbb{N}$, $k > k_0$, we take $U_k = I \times J_k$ where $J_k = [-2^{-k+1}, -2^{-k}] \cup$



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$[2^{-k}, 2^{-k+1}]$. So $W_1 = \overline{\cup U_k}$. For $(x_1, x_2) \in U_k$, it follows from (4.5) that

$$|\text{Hess}\varphi(x_1, x_2)| \geq c2^{-k(\frac{b}{2}-2)},$$

so for $\xi = (\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3$,

$$\left| \widehat{\sigma^{U_k}}(\xi) \right| \leq c2^{k\frac{b-4}{4}} (1 + |\xi_3|)^{-1}$$

and from Remark 3 we get

$$(4.6) \quad \|\mathcal{R}^{U_k}\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^2(\Sigma^{U_k})} \leq c2^{k\frac{b-4}{16}}.$$

Also, since $\left| \frac{\partial^2 \varphi}{\partial x_1^2}(x_1, x_2) \right| \geq c > 0$ uniformly on U_k , as in (2.4) we obtain

$$(4.7) \quad \|\mathcal{R}^{U_k}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{U_k})} \leq c2^{-k(2-\frac{2}{p})}$$

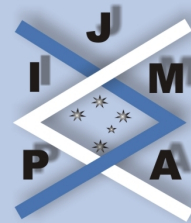
for $\frac{3}{4} < \frac{1}{p} \leq 1$ and $\frac{1}{q} = 3\left(1 - \frac{1}{p}\right)$. From (4.6), (4.7) and the Riesz Thorin theorem we obtain

$$(4.8) \quad \|\mathcal{R}^{U_k}\|_{L^{pt}(\mathbb{R}^3), L^{qt}(\Sigma^{U_k})} \leq c2^{k(t\frac{b-4}{16} - (1-t)(2-\frac{2}{p}))}$$

for $\frac{1}{qt} = t\frac{1}{2} + (1-t)3\left(1 - \frac{1}{p}\right)$ and $\frac{1}{pt} = t\frac{3}{4} + (1-t)\frac{1}{p}$.

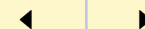
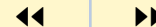
A simple computation shows that if $\frac{1}{p} = \frac{3}{4}$ then the exponent in (4.8) is negative for $t < t_0 = \frac{8}{4+b}$ and that

$$\frac{1}{qt_0} - \frac{2+3b}{4(2+b)} < 0,$$



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so for $\frac{1}{p} > \frac{3}{4}$ and $t < t_0$, both near enough, the exponent is still negative and

$$\frac{1}{q_t} - \frac{2+3b}{2+b} \left(1 - \frac{1}{p_t}\right) < 0,$$

thus

$$(4.9) \quad \|\mathcal{R}^{W_1}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{W_1})} < \infty$$

for $\frac{3}{4} < \frac{1}{p}$ near enough and $\frac{1}{q} = \frac{2+3b}{2+b} \left(1 - \frac{1}{p}\right)$. Finally, if

$$(B^2b^2 + 8ACb - 8ACb^2) (x_2^0)^{\frac{b}{2}} - 2(b-2)ABbx_1^0 = 0$$

then we study the order of $Hess\varphi(x_1, x_2^0)$ for $2^{-k-1} \leq |x_1 - x_1^0| \leq 2^{-k}$, $k \in \mathbb{N}$.

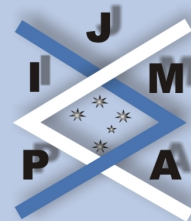
$$(4.10) \quad \left| \frac{(x_2^0)^{\frac{b}{2}-2}}{4} \left((B^2b^2 + 8ACb - 8ACb^2) (x_2^0)^{\frac{b}{2}} - 2(b-2)ABbx_1 \right) \right| \\ = \left| \frac{(x_2^0)^{\frac{b}{2}-2}}{2} (b-2)ABb(x_1 - x_1^0) \right| \geq c2^{-k}.$$

We take the following neighborhood of (x_1^0, x_2^0) , $W_2 = \overline{\cup_{k \in \mathbb{N}} V_k}$, with

$$V_k = \left\{ \left(r^{\frac{1}{2}}x_1, r^{\frac{1}{b}}x_2^0 \right) : 2^{-k-1} \leq |x_1 - x_1^0| \leq 2^{-k}, \frac{1}{2} \leq r \leq 2 \right\}.$$

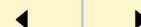
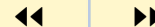
From the homogeneity of φ and (4.10) we obtain

$$\left| Hess\varphi \left(r^{\frac{1}{2}}x_1, r^{\frac{1}{b}}x_2^0 \right) \right| = r^{1-\frac{2}{b}} |Hess\varphi(x_1, x_2^0)| \geq c2^{-k},$$



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then from Proposition 6 in [8, p. 344], we get for $\xi = (\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3$

$$\left| \widehat{\sigma^{V_k}}(\xi) \right| \leq c 2^{\frac{k}{2}} (1 + |\xi_3|)^{-1},$$

so from Remark 3

$$\left\| \mathcal{R}^{V_k} \right\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^2(\Sigma^{V_k})} \leq c 2^{\frac{k}{8}}$$

and by Hölder's inequality, for $q < 2$ we have

$$\left\| \mathcal{R}^{V_k} \right\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^q(\Sigma^{V_k})} \leq c 2^{k \left(\frac{1}{8} - \frac{2-q}{2q} \right)}.$$

This exponent is negative for $\frac{1}{q} > \frac{5}{8}$ and so we sum on k to obtain

$$(4.11) \quad \left\| \mathcal{R}^{W_2} \right\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^q(\Sigma^{W_2})} < \infty$$

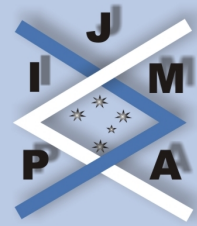
for $\frac{5}{8} < \frac{1}{q} \leq 1$. Since $b \geq 6$, $\frac{5}{8} \leq \frac{2+3b}{4(2+b)}$ and then from (4.1), (4.9) and (4.11), we get

$$\left\| \mathcal{R}^{A_0} \right\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{A_0})} < \infty,$$

for $\frac{3}{4} < \frac{1}{p}$ near enough and $\frac{1}{q} > \frac{2+3b}{2+b} \left(1 - \frac{1}{p} \right)$ and the theorem follows from standard considerations involving Hölder's inequality, the Riesz Thorin theorem and from Remark 2. \square

Remark 6. In the case $(a, b) = (2, b)$, $b > 2$, we have (4.11). In a similar way we get, from (4.6) and Hölder's inequality,

$$\left\| \mathcal{R}^{W_1} \right\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^q(\Sigma^{W_1})} < \infty$$



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for $\frac{b+4}{16} < \frac{1}{q} \leq 1$. So

$$\|\mathcal{R}\|_{L^{\frac{4}{3}}(\mathbb{R}^3), L^q(\Sigma)} < \infty$$

for $\max\left\{\frac{5}{8}, \frac{b+4}{16}, \frac{2+3b}{8+4b}\right\} < \frac{1}{q} \leq 1$. We observe that if $b = 6$ then $\frac{5}{8} = \frac{b+4}{16} = \frac{2+3b}{8+4b}$, thus from Remark 1 we see that, in this case, this condition for $\frac{1}{q}$ is sharp, up to the end point.

Now we will show some examples of functions φ not satisfying the hypothesis of the previous theorem, for which we obtain that the portion of the type set E in the region $\frac{3}{4} < \frac{1}{p} \leq 1$ is smaller than the region

$$E_{a,b} = \left\{ \left(\frac{1}{p}, \frac{1}{q} \right) : \frac{3}{4} < \frac{1}{p} \leq 1, \frac{a+b+ab}{a+b} \left(1 - \frac{1}{p} \right) < \frac{1}{q} \leq 1 \right\}$$

stated in Theorem 4.1.

We consider $\varphi(x_1, x_2) = x_1^2$, which is a mixed homogeneous function satisfying (1.1) for any $b > 2$. In this case $\varphi_{x_1x_1} \equiv 2$ but $Hess\varphi \equiv 0$. From Remark 2.8 in [4] and Remark 4 we obtain that the corresponding type set is the region $\frac{1}{q} \geq 3 \left(1 - \frac{1}{p} \right)$, $\frac{3}{4} < \frac{1}{p} \leq 1$ which is smaller than the region $E_{a,b}$.

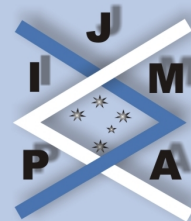
We consider now a mixed homogeneous function φ satisfying (1.1), of the form

$$(4.12) \quad \varphi(x_1, x_2) = x_2^l P(x_1, x_2),$$

with $P(x_1, 0) \neq 0$ for $x_1 \neq 0$. Since $a < b$ it can be checked that $l \geq 2$ and that for $l > 2$, $\varphi_{x_1x_1}(x_1, 0) = \varphi_{x_2x_2}(x_1, 0) = 0$. Moreover

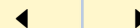
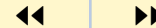
$$(4.13) \quad Hess\varphi = x_2^{2l-2} \left(P_{x_1x_1} \left(l(l-1)P + 2lx_2P_{x_2} + x_2^2P_{x_2x_2} \right) - (lP_{x_1} + x_2P_{x_1x_2})^2 \right),$$

which vanishes at $(x_1, 0)$. A computation shows that the second factor is different from zero at a point of the form $(x_1, 0)$. So $Hess\varphi$ does not vanish identically.



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Proposition 4.2. Let φ be a mixed homogeneous function satisfying (1.1) and (4.12). If $(\frac{1}{p}, \frac{1}{q}) \in E$ then $\frac{1}{q} \geq (l+1)(1 - \frac{1}{p})$.

Proof. Let $f_\varepsilon = \chi_{K_\varepsilon}$ the characteristic function of the set $K_\varepsilon = [0, \frac{1}{3}] \times [0, \frac{\varepsilon^{-1}}{3}] \times [0, \frac{\varepsilon^{-l}}{3M}]$, with $M = \max_{(x_1, x_2) \in [0, 1] \times [0, 1]} P(x_1, x_2)$. If $(\frac{1}{p}, \frac{1}{q}) \in E$ then

$$(4.14) \quad \|\mathcal{R}f_\varepsilon\|_{L^q(\Sigma)} \leq c \|f_\varepsilon\|_{L^p(\mathbb{R}^3)} = c\varepsilon^{-\frac{1+l}{p}}.$$

By the other side,

$$\|\mathcal{R}f_\varepsilon\|_{L^q(\Sigma)} \geq \left(\int_{W_\varepsilon} \left| \widehat{f}_\varepsilon(x_1, x_2, \varphi(x_1, x_2)) \right|^q dx_1 dx_2 \right)^{\frac{1}{q}}$$

where $W_\varepsilon = [\frac{1}{2}, 1] \times [0, \varepsilon]$. Now, for $(x_1, x_2) \in W_\varepsilon$ and $(y_1, y_2, y_3) \in K_\varepsilon$,

$$|x_1 y_1 + x_2 y_2 + \varphi(x_1, x_2) y_3| \leq 1$$

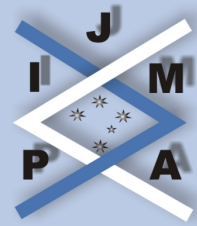
so

$$\begin{aligned} & \left| \widehat{f}_\varepsilon(x_1, x_2, \varphi(x_1, x_2)) \right| \\ &= \left| \int_{K_\varepsilon} e^{-i(x_1 y_1 + x_2 y_2 + \varphi(x_1, x_2) y_3)} dy_1 dy_2 dy_3 \right| \\ &\geq \int_{K_\varepsilon} \cos(x_1 y_1 + x_2 y_2 + \varphi(x_1, x_2) y_3) dy_1 dy_2 dy_3 \geq c\varepsilon^{-1-l}. \end{aligned}$$

Thus

$$(4.15) \quad \|\mathcal{R}f_\varepsilon\|_{L^q(\Sigma)} \geq c\varepsilon^{-1-l+\frac{1}{q}}.$$

The proposition follows from (4.14) and (4.15). □



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We note that in the case that $(a + b)l > ab$ (for example $\varphi(x_1, x_2) = x_2^4(x_1^2 + x_2^4)$) the portion of the type set corresponding to $\frac{3}{4} < \frac{1}{p} \leq 1$ will be smaller than the region $E_{a,b}$.

Also, $\varphi(x_1, x_2) = x_2^2(x_1 + x_2^2)$ is an example where $a = 2, b = 4, Hess\varphi(x_1, x_2) = -4x_2^2$ and if $x_2 = 0$ and $x_1 \neq 0$, $\varphi_{x_2x_2}(x_1, x_2) = 2x_1 \neq 0$. Again, since $12 = (a + b)l > ab = 8$, we get that the portion of the type set corresponding to $\frac{3}{4} < \frac{1}{p} \leq 1$ will be smaller than the region $E_{a,b}$.

Proposition 4.3. *Let φ be a mixed homogeneous function satisfying (1.1) and (4.12) with $l \geq \frac{b}{2}$. If $\frac{3}{4} \leq \frac{1}{p} \leq 1$ and $\frac{1}{q} > (l + 1)\left(1 - \frac{1}{p}\right)$, then*

$$\|\mathcal{R}^{A_0}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{A_0})} \leq c.$$

Proof. Let $(x_1^0, x_2^0) \in A_0$, if $Hess\varphi(x_1^0, x_2^0) \neq 0$, as in the proof of Theorem 4.1 we find a neighborhood U of (x_1^0, x_2^0) such that (4.1) holds. If $Hess\varphi(x_1^0, x_2^0) = 0$, by (4.13), either $x_2^0 = 0$ or the polynomial Q given by $P_{x_1x_1}(l(l-1)P + 2lx_2P_{x_2} + x_2^2P_{x_2x_2}) - (lP_{x_1} + x_2P_{x_1x_2})^2$ vanishes at (x_1^0, x_2^0) . In the first case, using the fact that $P(x_1, 0) \neq 0$ for $x_1 \neq 0$, we get that

$$(P_{x_1x_1}l(l-1)P - l^2P_{x_1}^2)(x_1^0, 0) \neq 0.$$

We take a neighborhood W_1 of the point $(x_1^0, 0)$ and U_k as in the proof of Theorem 4.1. So for $(x_1, x_2) \in U_k$,

$$|Hess\varphi(x_1, x_2)| \geq c2^{-k(2l-2)}$$

and so

$$\left| \widehat{\sigma^{U_k}}(\xi_1, \xi_2, \xi_3) \right| \leq \frac{2^{k(l-1)}}{1 + |\xi_3|}.$$



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By the other side,

$$\left| \widehat{\sigma^{U_k}}(\xi_1, \xi_2, \xi_3) \right| \leq 2^{-k}$$

so for $0 \leq \tau \leq 1$,

$$\left| \widehat{\sigma^{U_k}}(\xi_1, \xi_2, \xi_3) \right| \leq \frac{2^{k(\tau l - 1)}}{(1 + |\xi_3|)^\tau}$$

and by Remark 3

$$\|\mathcal{R}^{U_k}\|_{L^p(\mathbb{R}^3), L^2(\Sigma^{U_k})} \leq c_\tau 2^{\frac{k(\tau l - 1)}{2(1 + \tau)}}$$

for $p = \frac{2(1 + \tau)}{2 + \tau}$ and so Hölder's inequality implies, for $1 \leq q < 2$,

$$\|\mathcal{R}^{U_k}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{U_k})} \leq c_\tau 2^{k\left(\frac{\tau l - 1}{2(1 + \tau)} - \frac{2 - q}{2q}\right)}$$

and a computation shows that this exponent is negative for $\frac{1}{q} > (l + 1)\left(1 - \frac{1}{p}\right)$.

Thus

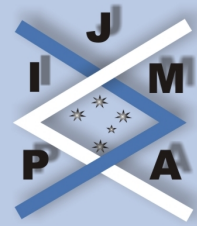
$$(4.16) \quad \|\mathcal{R}^{W_1}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{W_1})} < \infty$$

for $\frac{3}{4} \leq \frac{1}{p} \leq 1$ and $(l + 1)\left(1 - \frac{1}{p}\right) < \frac{1}{q} \leq 1$. Now we suppose $Q(x_1^0, x_2^0) = 0$. We observe that

$$\deg Q \leq 2 \deg P - 2 \leq 2(b - l) - 2 \leq 2l - 2$$

and so $Hess\varphi(x_1, x_2^0)$ vanishes at x_1^0 with order at most $2l - 2$. Then defining W_2 and V_k as in the proof of Theorem 4.1, we have

$$|Hess\varphi(x_1, x_2^0)| \geq 2^{-k(2l - 2)}$$



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and as in the previous case we obtain

$$(4.17) \quad \|\mathcal{R}^{W_2}\|_{L^p(\mathbb{R}^3), L^q(\Sigma^{W_2})} < \infty$$

for $\frac{3}{4} \leq \frac{1}{p} \leq 1$ and $\frac{1}{q} > (l+1)\left(1 - \frac{1}{p}\right)$. The proposition follows from (4.16), (4.17) and (4.1). \square

From Proposition 4.3 and Remark 2 we obtain the following result, sharp up to the end points, for $\frac{3}{4} \leq \frac{1}{p} \leq 1$.

Theorem 4.4. *Let φ be a mixed homogeneous function satisfying (1.1) and (4.12) with $l \geq \frac{b}{2}$. If $m = \max\left\{l+1, \frac{a+b+ab}{a+b}\right\}$, $\frac{3}{4} \leq \frac{1}{p} \leq 1$ and $\frac{1}{q} > m\left(1 - \frac{1}{p}\right)$, then $\left(\frac{1}{p}, \frac{1}{q}\right) \in E$.*

4.1. Sharp $L^p - L^2$ Estimates

In [4] we obtain sharp $L^p - L^2$ estimates for the restriction of the Fourier transform to homogeneous polynomial surfaces in \mathbb{R}^3 . The principal tools we used there were two Littlewood Paley decompositions. Adapting this proof to the setting of non isotropic dilations we obtain the following results.

Lemma 4.5. *Let $\frac{a+b+2ab}{2a+2b+2ab} \leq \frac{1}{p} \leq 1$. If*

$$\|\mathcal{R}^{A_0}\|_{L^p(\mathbb{R}^3), L^2(\Sigma^{A_0})} < \infty$$

then $\left(\frac{1}{p}, \frac{1}{2}\right) \in E$.

Proof. From (2.1), the lemma follows from a process analogous to the proof of Lemma 4.3 in [4]. \square



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Theorem 4.6.

- i) If φ is a mixed homogeneous polynomial function satisfying the hypothesis of Theorem 4.1 then $\left(\frac{a+b+2ab}{2a+2b+2ab}, \frac{1}{2}\right) \in E$.
- ii) Let $\frac{1}{p_0} = \max\left\{\frac{a+b+2ab}{2a+2b+2ab}, \frac{2l+1}{2l+2}\right\}$. If φ is a mixed homogeneous polynomial function satisfying the hypothesis of Theorem 4.4 then $\left(\frac{1}{p_0}, \frac{1}{2}\right) \in E$.

Proof. i) If $\frac{a+b+ab}{a+b} \geq 3$, i) follows from (4.3) and Lemma 4.5. The cases $(a, b) = (3, 4)$, $(a, b) = (3, 5)$ and $(a, b) = (4, 5)$ are solved in Remark 5, part ii). The cases $(a, b) = (2, b)$ with b odd or $B = 0$ are also included in Remark 5, part ii). For the remainder cases $(2, b)$, we observe that, if $b > 6$, from the proof of Theorem 4.1 we obtain

$$(4.18) \quad \|\mathcal{R}^{A_0}\|_{L^p(\mathbb{R}^3), L^2(\Sigma^{A_0})} < \infty,$$

for $\frac{1}{p} = \frac{a+b+2ab}{2a+2b+2ab}$, so i) follows from Lemma 4.5. For $b = 6$, as before we get

$$\|\mathcal{R}^{W_1}\|_{L^p(\mathbb{R}^3), L^2(\Sigma^{W_1})} < \infty,$$

and

$$\|\mathcal{R}^{V_k}\|_{L^p(\mathbb{R}^3), L^2(\Sigma^{V_k})} < \infty$$

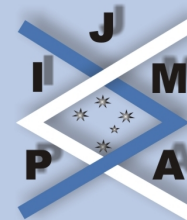
for $k \in \mathbb{N}$, $\frac{1}{p} = \frac{a+b+2ab}{2a+2b+2ab}$. In a similar way to Lemma 4.3 of [4], we use a uni-dimensional Littlewood Paley decomposition to obtain

$$\|\mathcal{R}^{W_2}\|_{L^p(\mathbb{R}^3), L^2(\Sigma^{W_2})} < \infty$$

and then we have (4.18) for $\frac{1}{p} = \frac{a+b+2ab}{2a+2b+2ab}$. So i) follows from Lemma 4.5.

ii) From the proof of Proposition 4.3, we use a uni-dimensional Littlewood Paley decomposition to obtain (4.18) for $\frac{1}{p} = \max \left\{ \frac{a+b+2ab}{2a+2b+2ab}, \frac{2l+1}{2l+2} \right\}$, and *ii)* follows from Lemma 4.5. \square

Remark 7. In [7] the authors obtain sharp estimates for the Fourier transform of measures σ associated to surfaces Σ like ours, when φ is a polynomial function satisfying (1.1) and the condition that φ and $Hess\varphi$ do not vanish simultaneously on $B - \{(0, 0)\}$. In these cases, part *i)* of the above theorem follows from Remark 3. We observe that our hypotheses are less restrictive, for example $\varphi(x_1, x_2) = x_1^4 x_2^2 + x_2^{10}$ satisfies the hypothesis of part *i)* of the above theorem but φ and $Hess\varphi$ vanish at any (x_1, x_2) with $x_2 = 0$.



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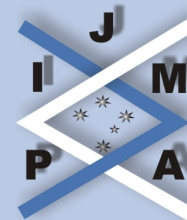
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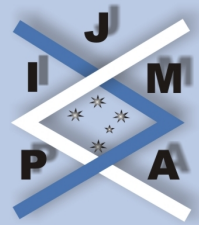
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Fourier Restriction
Estimates

E. Ferreyra and M. Urciuolo

vol. 10, iss. 2, art. 35, 2009

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