NONLINEAR FILTERING FOR LIDAR SIGNAL PROCESSING

D. G. LAINIOTIS*

Intelligent Systems Technology, 3207 S. Hwy. A1A, Melbourne Beach, FL 32951

PARASKEVAS PAPAPARASKEVA†

Stanford Telecom 1221 Crossman Av. Sunnyvoile CA 94089 Tel: 408-730-5062. Email: uakis-Pdstelg.com

KOSTAS PLATANIOTIS‡

Department of Electrical Engineering, University of Toronto, Toronto, Canada, 24 Wellesley St. West, Apt. 402, M4Y 1G, Toronto, Canada

LIDAR (Laser Integrated Radar) is an engineering problem of great practical importance in environmental monitoring sciences. Signal processing for LIDAR applications involves highly nonlinear models and consequently nonlinear filtering. Optimal nonlinear filters, however, are practically unrealizable. In this paper, the Lainiotis's multi-model partitioning methodology and the related approximate but effective nonlinear filtering algorithms are reviewed and applied to LIDAR signal processing. Extensive simulation and performance evaluation of the multi-model partitioning approach and its application to LIDAR signal processing shows that the nonlinear partitioning methods are very effective and significantly superior to the nonlinear extended Kalman filter (EKF), which has been the standard nonlinear filter in past engineering applications.

Keywords: LIDAR estimation, remote sensing, adaptive filtering, nonlinear filtering, multi-model partitioning, Lainiotis filters, extended Kalman filter

1. INTRODUCTION

The remote sensing of atmospheric and oceanic properties in both active and passive models has been traditionally limited due to the nature of the classical instrumentation available. The insufficient penetration of infrared and microwave radiation, especially through the water, has restricted most of the oceanic studies to surface characteristics that often do not provide one with a complete and informative picture concerning the distribution of features with depth. Among alternate observation procedures currently

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*Address for correspondence: D. G. Lainiotis, Intelligent Systems Technology, 3207 S. Hwy. A1A, Melbourne Beach, FL 32951, Tel/Fax: 407-768-1990

†Tel: 407-984-5164. Email: parask@ee.fit.edu

‡Tel: 416-969-9368. Email: kostas@dsp.toronto.edu

available, the most viable method is that of obtaining vertical profiles of radar-like range gated systems utilizing lasers as the radiation emitting source. Such laser systems are referred to as LIDAR [40]–[45]. They operate near appropriate wavelengths and have sufficient penetration of the order of tens of meters under favorable conditions. LIDAR systems are usually installed and operate from aircraft or space, and they provide indispensable data to both oceanographic and climatic studies.

Typically, atmospheric parameter estimation has entailed signal processing techniques that are based on single-pulse LIDAR returns. For example, in many LIDAR return cases, a simple estimator such as the average is adequate [12], [36], [37], [50]. Recently, the advantages of LIDAR estimate processing over multiple pulses have been recognized and consequently addressed extensively. In their pioneering work [41], [42], [44], Rye and Hardesty have applied the state-variable formulation and the related linear Kalman filter (KF) [14], and the nonlinear extended Kalman filter (EFK) [7], [10] to the estimation of the return power and to the logarithm of the return power for incoherent backscatter LIDAR in which multiplicative noise or speckle is present. The results obtained by Rye and Hardesty have been extended by the authors to the case of unknown LIDAR state-variable models [30], [31]. Several drawbacks are usually associated with LIDAR state-variable models. The whiteness assumption of the Kalman theory is violated, as the multiplicative noise source, seen in the speckle, exhibits serial correlation, and, in addition, the same quantity demonstrates non-Gaussian statistics. In addition, the environment around the measurement may abruptly introduce unknown bias effects in the observation sequence, or, from the system's reference point, random failures may suddenly occur. Finally, system parameters or part of the signal structure is usually unknown, which requires adaptive filter designs for their determination [48]. The study described in [42] has eliminated some of the major difficulties discussed above by the use of the optimization process of an adaptive simplex routine [38]. Reiterated passes are typically made through the data to jointly evaluate the signal power and track the uncertain parameters in the unmodeled dynamics. The real-time implementation of such an approach, nevertheless, allows for the possible introduction of significant delays during the filter realization.

This work reviews efficient methodologies for the effective estimation of the logarithm of the LIDAR return power in which multiplicative noise or speckle is present and for which the state-variable model is unknown. The paper is organized as follows: In section II, the form of a possible LIDAR system model in the presence of both additive and multiplicative noise (speckle) is presented with particular reference to estimation of the log power returns. Section III addresses the optimal solution of the general nonlinear estimation problem, explains the implementation difficulties that accompany the exact realization of the formulation, and provides some necessary simplifying assumptions to facilitate the introduction of practical approximate filters. Section IV develops the algorithmic basis for the Lainiotis partitioned adaptive filter as a parallel implementation of a bank of EKFs. The actual filter design for the specific LIDAR uncertain model of interest is outlined in Section V. Section VI reviews the results of extensive simulation studies on the performance of the alternate filters and gives a comprehensive error comparison between the partitioned algorithms and the classical EKF. Section VII summarizes significant findings from the study and provides a series of concluding remarks.

2. LIDAR SIGNAL PROCESSING: NONLINEAR ESTIMATION PROBLEMS

A. Measurement Equation

The development here utilizes the state-variable approach to model the system of interest. The desired quantity to be estimated is denoted by x_1 (k) and corresponds to the kth discrete sample of LIDAR signal power return received at the sensor. The state return is assumed to arrive corrupted by additive Gaussian noise, v (k), with zero mean and covariance R (k). The measurement equation takes then the form

$$z\,(k) = \Theta_z x_1\,(k) + v\,(k), \tag{1}$$

where Θ_z represents an unknown parameter to be identified. This is introduced to account for the fact that the designer may have no knowledge of how the returns are scaled and appear at the sensor that receives the readings. To estimate the absorbance of the LIDAR channel, which is proportional to the logarithm of the return power, the measurement equation becomes

$$z\,(k) = \Theta_z exp\,[x_1\,(k)] + v\,(k), \tag{2}$$

with Θ_z having a similar meaning as before. Moreover, if one includes a source of speckle or multiplicative noise, equation (2) transforms to

$$z\,(k) = \Theta_z\,W\,(k)\,exp\,[x_1\,(k)] + v\,(k), \tag{3}$$

where W (k) is the speckle term just mentioned. Research studies have shown that, spectrally, W (k) should be expected to be uncorrelated from pulse to pulse, and, statistically, its probability density function may be nonsymmetrical, although approximately Gaussian for high-order speckle [42]. Other properties required are that W (k) should be positive, on the average equal to unity, and expressible in terms of Gaussian sequences to comply with Kalman theory stipulations. A simple model to satisfy the above observations and speckle characteristics is given by

$$W\,(k) = 1 + w\,(k), \tag{4}$$

where w (k) is a zero mean Gaussian noise sequence [42].

B. System Equations

To simultaneously keep track of signal power returns and speckle perturbation, a two-dimensional state space representation is adopted as follows:

$$x_1\,(k+1) = x_1\,(k) + \Theta_w w_1\,(k), \tag{5}$$

$$x_2\,(k+1) = 1 + w_2\,(k), \tag{6}$$

where the first state, $x_1(k)$, is the power return and the second, $x_2(k)$, is the speckle; $w_1(k)$ is a white Gaussian sequence with zero mean and unity variance that is scaled to $Q_1(k)$ by the unknown parameter Θ_w; and $w_2(k)$ is the zero mean white Gaussian sequence of equation (4), independent from $w_1(k)$, having covariance $Q_2(k)$. In meteorological measurements, the strength of the additive stochastic disturbance associated with equation (5) is unknown, and hence the purpose of the quantity Θ_w becomes meaningful and effective. The measurement equation may be rewritten to accommodate the state dynamics format as

$$z(k+1) = \Theta_z x_2(k+1) \, exp \, [x_1(k+1)] + v(k+1). \tag{7}$$

Expressions (5)–(7) are finally in a form which permits discrete nonlinear state estimation and identification. Future simulation experiments have fallen into two major categories. The first involves the state dependent unknown noise parameter Θ_w of the plant, whereas the other one deals with the uncertainty connected to Θ_z in the measurement returns. In general, the optimal nonlinear estimator is known to be practically unrealizable as shown in the next section [1], [21], [46], [47]. Effective approximate formulations, nonetheless, exist and are also reviewed in the sequence.

3. OPTIMAL NONLINEAR ESTIMATION

The problem of estimating the parameters and/or states of a nonlinear system—whether the nonlinearity is inherent to the model generating the stochastic process or is introduced by the observation mechanism—is a truly complicated one. Optimal nonlinear filtering is far less precise than its linear counterpart and one has to work hard to achieve even little. In the study of decision theory, nevertheless, one senses the need for nonlinear filtering algorithms. Many frequency and phase modulation problems, for example, have nonlinear observation structures, and the dynamic message models for the majority of realistic vehicle-guidance and control systems are by nature nonlinear. Signal processing nonlinear situations are also abundant, namely, the typical LIDAR formulation of this study. As a result of the practical necessity for solutions to such problems, many ideas and procedures have been proposed and investigated in the literature. Although some of them are no more than a philosophy of approach, rather than a procedure leading to derivation of practical estimators, there are a few that attack specific problems and result in useful filtering formulations satisfying the limited objectives.

In general, one has to accept that, when dealing with nonlinear dynamical system estimation, an analytical solution in closed form is not likely to be available and, instead, computational algorithms should be sought in their place [2]. The following subsections introduce the statement of the general nonlinear problem, outline the form of the optimal solution, and explain the implementation difficulties that lead to the search for alternative, suboptimal configurations. Such approximations are reviewed in later sections and include extended Kalman and Lainiotis partitioning filters.

A. Nonlinear Dynamical Model Description

The general state space model for a discrete time nonlinear stochastic system has the following form [7], [9]: System model:

$$x(k + 1) = f(k, x(k), w(k)). \tag{8}$$

Measurement model:

$$z(k) = h(k, x(k), v(k)). \tag{9}$$

If additive Gaussian excitation and measurement noise is assumed, the above model can be rewritten as a simpler model with the form given next. System model:

$$x(k + 1) = f(x(k), k) + w(k). \tag{10}$$

Measurement model:

$$z(k) = h(x(k), k) + v(k). \tag{11}$$

In above notation, $f(\ldots)$ is a nonlinear function of the states that depends on the index, k, $w(k)$ is zero mean, Gaussian noise having variance $Q(k)$, $h(\ldots)$ is a nonlinear function of the states that depends on the index, k, and $v(k)$ is zero mean, Gaussian noise having variance, $R(k)$. The following figure gives a block diagram of the nonlinear prototype system.

Comments

- The state vector x of the system evolves according to a nonlinear stochastic difference equation, in which the vector valued function $f(\ldots)$ is in general time-varying.

- The initial state of the system, $x(0)$, is assumed to be described by a known probability density function (pdf).

- The behavior of the plant is observed imperfectly through the nonlinear, stochastic, time-varying function $h(\ldots)$.

The objective of the optimal nonlinear estimator is to obtain the optimal estimates of the stochastically varying state vector $X^k = \{x(1), x(2),\ldots,x(k)\}$, given the available information contained in the related sequence of measurement vectors $Z^k = \{z(1), z(2),\ldots,z(k)\}$ [10].

B. Nonlinear Estimator Equations

The best representation of the system's states at a particular time instant k is provided by the conditional probability density function of the state vector $x(k)$, given all pertinent information available at time k; such information involves the initial state of the system

and all past measurements through time k. If the probability of interest, denoted $p\,(x\,(k)|Z^k)$, is somehow available, one can obtain a number of estimates of the desired state vector $x\,(k)$ according to prespecified criteria. These estimates are usually a consequence of a straightforward computational manipulation of the density function [1], [47].

To begin, one assumes that the initial probability density function, $p\,(x\,(0)|z\,(0))$ is computable from the a-priori information of the state space equations (8) and (9), and that the probability density functions $p\,(x\,(k+1)|X^k)$ and $p\,(z\,(k)|X^{k-1})$ can be obtained from those of $w\,(k)$ and $v\,(k)$. Using the familiar Bayes' rule theorem, the following relationships hold:

$$p\,(X^k|Z^k) = \frac{p\,(X^k, Z^k)}{p\,(Z^k)}, \tag{12}$$

with

$$p\,(X^k, Z^k) = p\,(x\,(k), X^{k-1}, z\,(k), Z^{k-1}), \tag{13}$$

or, in terms of more elementary probability densities,

$$p\,(X^k, Z^k) = p\,(Z^{k-1})\,p\,(X^{k-1}|Z^{k-1})\,p\,(x\,(k)|X^{k-1}, Z^{k-1})\,p\,(z\,(k)|X^k, Z^{k-1}), \tag{14}$$

and

$$p\,(Z^k) = p\,(z\,(k)\,|\,Z^{k-1})\,p\,(Z^{k-1}), \tag{15}$$

so that equation (12) becomes

$$p\,(X^k|Z^k) = \frac{p\,(X^{k-1}|Z^{k-1})\,p\,(x\,(k)\,|X^{k-1}, Z^{k-1})\,p\,(z\,(k)|\,X^k, Z^{k-1})}{p\,(z\,(k)\,|\,Z^{k-1})}, \tag{16}$$

and the denominator is given by

$$p\,(z\,(k)|Z^{k-1}) = \int p\,(X^{k-1}|Z^{k-1})\,p\,(x\,(k)\,|X^{k-1}, Z^{k-1})\,p\,(z\,(k)|\,X^k, Z^{k-1})\,dX^k, \tag{17}$$

where $dX^k = dx\,(1)\,dx\,(2)...dx\,(k)$.

If the noise sequences are assumed (conditioned on the state vector $x\,(k)$) mutually independent, additive, white noises as in the simpler case of equations (10) and (11), then the optimal estimator can be determined using a recursive functional relationship among a-posteriori densities as follows:

$$p\,(x\,(k)|Z^k) = c_k p\,(x\,(k)\,|Z^{k-1})\,p\,(z\,(k)|\,x\,(k), Z^{k-1}), \tag{18}$$

$$p\left(x\left(k\right)|Z^{k-1}\right) = \int p\left(x\left(k\right)|x\left(k-1\right), Z^{k-1}\right) p\left(x\left(k-1\right)|Z^{k-1}\right) dx\left(k-1\right), \quad (19)$$

and

$$\frac{1}{c_k} = \int p\left(x\left(k\right)|Z^{k-1}\right) p\left(z\left(k\right)|x\left(k\right), Z^{k-1}\right) dx\left(k\right), \quad (20)$$

where Z^k is the record of all the measurements up to and including time k.

Comments

- The cumulative measurement set Z^k is the complete substitute for the past data in the probability density function for any present and future quantity related to the system. The same holds true for the state set X^k.

- For a white, independent measurement noise sequence, the following equation is true:

$$p\left(z\left(k\right)|x\left(k\right) = p\left(z\left(k\right)|X^k, Z^{k-1}\right). \quad (21)$$

- If the process noise is also a white, independent sequence and in addition is mutually uncorrelated with the measurement noise, then the state vector $x\left(k\right)$ constitutes a Markov process satisfying

$$p\left(x\left(k\right)|x\left(k-1\right)\right) = p\left(x\left(k\right)|X^{k-1}, Z^{k-1}\right). \quad (22)$$

- Based on the previous ideas, the state prediction probability density function is given by the Chapman-Kolmogorov equation:

$$p\left(x\left(k\right)|Z^{k-1}\right) = \int p\left(x\left(k\right)|x\left(k-1\right)\right) p\left(x\left(k-1\right)|Z^{k-1}\right) dx\left(k-1\right). \quad (23)$$

- The joint probability density function of the measurements up to the time instant k is given by:

$$p\left(Z^k\right) = p\left(z\left(k\right), Z^{k-1}\right) = p\left(z\left(k\right)|Z^{k-1}\right) p\left(Z^{k-1}\right) = \prod_{i=1}^{k} p\left(z\left(i\right)|Z^{i-1}\right). \quad (24)$$

- The above joint density of the observation record, which is conditioned indirectly on the system model, forms the likelihood function of the system's model. The likelihood function serves as a normalizing factor in the recursive calculations of the state a-posteriori probability density function.

Given that the probability density of the initial condition, $p\left(x\left(0\right)\right)$, is known a-priori, or assumed to have a specific form, equations (18)–(20) constitute a nonlinear functional recursion, which enables one to obtain the probability density function $p\left(x\left(k\right) | Z^k\right)$ at time k from $p\left(x\left(k-1\right) | Z^{k-1}\right)$ and the new measurement $z\left(k\right)$.

C. Implementation Considerations

The recursion scheme outlined in the previous subsection seems promising and powerful; the probability densities involved, however, are not Gaussian and, as a result, they cannot be completely described from the first two moments (or in general from any finite number of moments). Thus, the actual realization of the optimal nonlinear estimator is hampered by a series of implementation difficulties. Such considerations are stated next.

- Memory requirements: The storage of the probability density function used in the recursive nonlinear equations is, in general, equivalent to an infinite-dimensional vector.

- Computational requirements: The integrals involved in the recursion are not likely to have a closed form solution; as such, the integrations have to be evaluated numerically with calculations that are routine but represent tedious and time-consuming operations.

With the exception of the linear Gaussian models, the above functional recursive formulation is apparently impractical for real nonlinear estimation problems. On the other hand, if the system is linear and the disturbances are assumed Gaussian, only a finite statistic, consisting of the state's mean and error covariance, is sufficient to implement the equations (18)–(20). The Kalman [14], Lainiotis [25], or any other linear optimal filter provide the means for a recursive update of such a sufficient statistic.

D. Approximations of the Optimal Nonlinear Estimator

The implementation difficulties associated with a truly optimal nonlinear estimator, on one hand, along with the practical importance that accompanies the solution of nonlinear filtering problems, on the other hand, have led to the development of approximation procedures that facilitate the determination of the states in nonlinear stochastic systems. Such approximate methodologies introduce an estimation criterion that helps reduce the available information concerning the state to a finite collection of numbers [1]. The most commonly used criterion minimizes the quadratic performance index $J(k)$ defined as

$$J(k) = E\{[x(k) - g(Z^k)]^T Q_j [x(k) - g(Z^k)]\}, \qquad (25)$$

where Q_j is a positive definite matrix. For example, if $Q_j = I$, then the index becomes

$$J(k) = E\{[x(k) - g(Z^k)]^T [x(k) - g(Z^k)]\}. \qquad (26)$$

Other fundamental assumptions behind approximating scenarios is that the involved a-priori distributions are Gaussian and that the nonlinear state space model can be linearized with respect to prespecified reference values.

Given the mentioned stipulations, one can now construct a series of practical, yet suboptimal, non-linear estimation algorithms. Linearized, extended Kalman, and Lainiotis partitioning filters, along with some of their modified versions, fall into this category [7], [24], [28]. The following sections are intended to highlight some of the important aspects underlying these techniques. As expected, such approaches do not necessarily ensure high

reliability or robustness, as they create a mismatch between the approximate linear framework of the procedure and the inherent nonlinear structure of the actual model.

4. PRACTICAL NONLINEAR ESTIMATORS: EXTENDED KALMAN FILTER AND EFFECTIVE PARTITIONING NONLINEAR FILTERS

A. The Extended Kalman Filter

So as not to depart very far from the standard linear Gaussian model, one usually considers the simpler nonlinear model described by equations (10) and (11). Imposing some mathematical formalism on the nonlinearities, such as continuity, smoothness, and differentiability, the functions $f(x(k), k)$ and $h(x(k), k)$ may be expanded in a Taylor series about the conditional means $\hat{x}(k|k)$ and $\hat{x}(k|k-1)$ as follows [7], [10]:

$$f(x(k), k) = f(\hat{x}(k|k), k) + F(k)(x(k) - \hat{x}(k|k)) + \cdots, \tag{27}$$

$$h(x(k), k) = h(\hat{x}(k|k-1), k) + H(k)(x(k) - \hat{x}(k|k-1)) + \cdots, \tag{28}$$

Neglecting higher order terms and assuming that the quantities $\hat{x}(k|k)$ and $\hat{x}(k|k-1)$ have already been processed and thus are known, the nonlinear model of equations (10) and (11) can be approximated as

$$x(k+1) = F(k)x(k) + w(k) + u(k), \tag{29}$$

$$z(k) = H(k)x(k) + v(k) + y(k), \tag{30}$$

where $u(k)$ and $y(k)$ are calculated on-line from

$$u(k) = f(\hat{x}(k|k), k) - F(k)\hat{x}(k|k), \tag{31}$$

$$y(k) = h(\hat{x}(k|k-1), k) - H(k)\hat{x}(k|k-1), \tag{32}$$

and $F(k)$, $H(k)$ are given by the following expressions:

$$F(k) \equiv \frac{\partial}{\partial x(k)} f(x(k), k) \Big|_{x(k) = \hat{x}(k/k)}, \tag{33}$$

$$H(k) \equiv \frac{\partial}{\partial x(k)} h(x(k), k) \Big|_{x(k) = \hat{x}(k/k-1)} \tag{34}$$

Once transformed into the above approximate signal model, the extended Kalman filter (EKF) for the original nonlinear system is simply a trivial variation of the standard Kalman filter algorithm [14]. It must be noted, however, that the notations $\hat{x}(k|k-1)$ and

P ($k|k - 1$) are loosely used and denote only approximate conditional means and error covariances of the true process $x(k)$. The recursive equations for the discrete extended Kalman filter are given below.

Initial conditions:

$$x\ (0) \sim N(\hat{x}(0|0), P\ (0|0)). \tag{35}$$

Other assumptions:

$$cov\ \{w\ (k), v\ (j)\} = 0\ \text{for all } k, j. \tag{36}$$

State extimation propagation:

$$\hat{x}\ (k + 1|k) = f\ (\hat{x}\ (k|k), k). \tag{37}$$

Error covariance propagation:

$$P\ (k + 1|k) = F\ (k)P\ (k|k)F^T\ (k) + Q\ (k). \tag{38}$$

Filter gain:

$$K\ (k + 1) = P\ (k + 1|k)\ H^T\ (k + 1)\ [H\ (k + 1)P\ (k + 1|k)H^T\ (k + 1) + R\ (k + 1)]^{-1}. \tag{39}$$

State estimate update:

$$\hat{x}(k + 1|k + 1) = \hat{x}(k + 1|k) + K\ (k + 1)\ [z\ (k +) - h\ (\hat{x}(k + 1|k), k + 1)]. \tag{40}$$

Error covariance update:

$$P(k + 1|k + 1) = [I - K(k +)H(k + 1)]P(k + 1|k), \tag{41}$$

where $F(k)$ and $H(k + 1)$ are adjusted as in equations (33) and (34).

Comments

- The filer gain $K(k)$ and the approximate performance measure $P(k|k - 1)$ are coupled to the filter equations since they depend on the linearizing reference point of $\hat{x}(k|k - 1)$. As a result, one concludes that in general the calculation of $K(k)$ and $P(k|k - 1)$ cannot be carried out off-line. If, however, the application at hand necessitates a-priori computation of these quantities, then a deterministic reference trajectory may be used in the linearization process, in which case one deals with the so-called linearized Kalman filter [7], [10]. In practice, the reference trajectory is obtained by developing an

appropriate mathematical model of the process in question or by simulating about some reasonable operating conditions with respect to the actual system.

- The quality of the approximation involved in passing from the original nonlinear system of equations (10), (11) to that of (29), (30) improves if the quantities $||x(k) - \hat{x}(k|k)||^2$ and $||x(k) - \hat{x}(k|k - 1)||^2$ are smaller. Thus, in high signal-to-noise ratio situations, one would expect fewer difficulties and better performance in using the EKF. Another option one may consider in determining whether or not the filter will perform adequately is to check the degree of whiteness of the pseudo-innovations, for the whiter these are the more nearly optimal the filter becomes.

- Variations of the EKF have also been considered in literature by slightly altering the derivation procedures and the assumptions involved in the derivation. Such modifications include higher order filters, where more terms are kept in the Taylor series expansion, and iterated EKFs, where the reference trajectory can be improved by iteration techniques. Although both methods aim to better their respective estimates, they differ in their philosophies for accomplishing their common objective. The former seeks a better approximation to the optimal filter, whose structure is constrained by the fact that the measurement appears linearly in the state estimate; the latter allows a more general dependence of the estimate upon the observation data. Application of the EKF in engineering problems has been very common and popular [3], [4], [8]. A comparative analysis of several related nonlinear techniques has been reported by Mehra [35] and Wishner et al. [49].

The minimum-variance-based estimation, with the Taylor expansion approximation, is by no means the only possibility when designing nonlinear filters. Several other alternatives have been suggested in the literature, and they may be superior to the EKF related algorithms in particular problems. A statistical linearization found in [10], for instance, has shown overall better performance than the Taylor series based approaches. Maximum a-posteriori probability (MAP) and nonlinear least-squares estimation criteria have also been employed in several places as a substitute for minimum variance techniques [6], [39]; the latter is especially useful in situations where the statistical properties of the uncertain quantities are not well-defined or not even known at all. Yet another class of nonlinear estimators depends upon finding functional approximations of the conditional probability density function of the state x. Details can be found in Sorenson [46], [47].

B. Adaptive Estimation: The Lainiotis Partitioning Theory

In many physical phenomena, one is often confronted with the task of designing an estimator in the face of incomplete model knowledge. In such cases, the approach usually taken is to implement the filter in a way that offers self-learning capabilities, so that it can adapt itself to the particular environment at hand. The discussion here primarily focuses on the development of methods and procedures for system adaptation or identification in the presence of parametric or structural uncertainties. The subsections that follow formulate the problem for the linear case and present the Lainiotis adaptive multi-model approach. The results are subsequently generalized to include a treatment of the nonlinear

scenario. Finally, state augmentation techniques for nonlinear system adaptation are described as an alternative approach to parallel-structured methods.

Adaptive estimation problems may be categorized according to:

- The nature of the basic mathematical model, e.g., linear, nonlinear, etc.

- The nature of the model uncertainty, i.e., parametric versus functional or structural uncertainty, time-invariant versus time-varying.

- The nature of the constraints imposed on the adaptation process.

The basic element in the design of adaptive estimation filters consists of linear models with time-invariant parametric uncertainties. More elaborate schemes may also be treated by appropriately modifying the standard problem; such extensions incorporate time-varying unknown parameters or structures and are to be reported elsewhere. It should be noted, however, that, even in the simplest possible case of linear Gaussian state space models, because of the introduced uncertainty, the problem becomes one of nonlinear estimation. The exact solution to the described problem is credited to Lainiotis [17]–[28], who was able to decompose the original nonlinear problem into a bank of linear subparts in a process known as the adaptive multi-model algorithm [18], [20], [26]–[28].

Apart from the multi-model methodology, which is the only optimal solution but could well be too complex for consideration, an ensemble of other suboptimal results have been made available in various sources. Adaptive estimation via least-squares, for example, is a popular choice with many specializations [16], [34]. The Kalman filter as an identifier has also been employed by Ljung and Soderstrom [33] and Goodwin [11] in certain applications. Augmented state models, which incorporate the unknown parameters and account for their estimation, is the only suboptimal technique to be introduced here; such methods can be used with the usual linear or nonlinear filters to perform state estimation and identification at the same time. Theoretical justifications and convergence analysis for this type of adaptation has been given by Ljung [32].

C. Linear Uncertain Models

A linear scenario is first treated. The adaptive estimation problem considered is specified by the following equations (linear Gaussian uncertain model):

$$x (k + 1) = \Phi (k + 1,k;\Theta)x (k) + \Gamma (k;\Theta)w (k), \qquad (42)$$

$$z (k + 1) = H (k + 1;\Theta)x (k + 1) + v (k + 1), \qquad (43)$$

where $x(k)$ is the state vector of the system, $\Phi(k + 1,k;\Theta)$ is the (possibly unknown) transition matrix, $\Gamma(k;\Theta)$ is the (possibly unknown) standard deviation of the noise term $w(k)$, $z(k)$ is the measurement vector, $H(k;\Theta)$ defines the observation matrix that may contain uncertainties, and $v(k)$ is the additive noise that corrupts the measurements. The unknown parameters are denoted by the vector Θ, which, if known, would completely specify the model. Moreover, Θ is considered to be a random variable with known or assumed a-priori density $p(\Theta|0) = p(\Theta)$. The processes $w(k)$ and $v(k)$ are still uncorrelated

when conditioned on Θ, with covariances $Q(k;\Theta)$ and $R(k;\Theta)$, respectively. The initial state vector $x(0)$ is conditionally Gaussian for given Θ, with mean $\hat{x}(0|0;\Theta)$ and covariance $P(0|0;\Theta)$, and is conditionally uncorrelated with the sequences $w(k)$ and $v(k)$.

The above partially unknown model is specified up to the unknown parameter vector, Θ, which may be time-invariant or time-varying. The time invariance of the parameter vector Θ may be justified on the basis of physical considerations or simply as an approximation to slowly time-varying parameters. Indeed, usually, as pointed out by Lainiotis [18], the parameters have much longer time constants than the actual system states. Given the measurement record $\lambda_k = \{z(1), z(2),\ldots, z(k)\}$, the objective is to obtain the optimal, in the minimum mean-square-error (mmse) sense, state estimate $\hat{x}(k|k)$ of $x(k)$.

D. Lainiotis's Multi-Model Approach: Adaptive Lainiotis Filters

Under the model conditions of equations (42) and (43), the optimal mean-square estimate $\hat{x}(k|k)$ of $x(k)$ and the corresponding error covariance matrix $P(k|k)$ are given by the following expressions [18], [26]:

$$\hat{x}\,(k|k) = \int \hat{x}\,(k|k;\Theta)p\,(\Theta|k)d\Theta, \tag{44}$$

$$P\,(k|k) = \int \{P\,(k|k;\Theta) + [\hat{x}\,(k|k) - \hat{x}\,(k|k;\Theta)]\,[\hat{x}\,(k|k) - \hat{x}\,(k|k;\Theta)]^{T}\}p\,(\Theta|k)d\Theta, \tag{45}$$

where $\hat{x}(k|k;\Theta)$ and $P(k|k;\Theta)$ are the Θ-conditional state vector estimate and the corresponding Θ-conditional error covariance matrix, and they are obtained from the corresponding filter matched to the model with parameter value Θ and initialized to $\hat{x}(0|0;\Theta)$ and $P(0|0;\Theta)$, respectively.

The a-posteriori probability density function $p(\Theta|k)$, given the record λ_k, can be computed by the following recursive Bayes-rule formula, Lainiotis [22], [23], [26]:

$$p\,(\Theta|k) = \frac{L\,(k|k;\Theta)}{\int L\,(k|k;\Theta)p\,(\Theta|k-1)d\Theta}\,p\,(\Theta|k-1), \tag{46}$$

where

$$L\,(k|k;\Theta) = |P_{z}\,(k|k-1;\Theta)|^{-1/2}exp[-\frac{1}{2}\|\tilde{z}\,(k|k-1;\Theta)\|^{2}P_{z}^{-1}\,(k|k-1;\Theta)], \tag{47}$$

and $\tilde{z}(k|k-1;\Theta)$ and $P_z(k|k-1;\Theta)$ are obtained from the filter matched to the model with parameter value Θ, namely,

$$\tilde{z}\,(k|k-1;\Theta) = z\,(k) - H\,(k;\Theta)\hat{x}\,(k|k-1;\Theta), \tag{48}$$

$$P_{z}\,(k|k-1;\Theta) = H\,(k;\Theta)P\,(k|k-1)H\,(K;\Theta)^{T} + R\,(k;\Theta). \tag{49}$$

Remarks

- A feature of cardinal importance in the above partitioned realization is that the optimal mmse estimate $\hat{x}(k|k)$ and its error covariance matrix $P(k|k)$ are given in terms of the model-conditional estimates $\hat{x}(k|k;\Theta)$ and the corresponding model-conditional covariance matrices $P(k|k;\Theta)$ in a weighted-sum, decoupled, parallel-structure form, which is convenient and natural for parallel processing.

- Partitioning, through conditioning on a pivotal set of vector parameters Θ, decomposes or disaggregates a complex or large scale estimation problem into a bank of simple subparts, which are non-adaptive and linear, and a nonlinear part, consisting of the a-posteriori probability density function $p(\Theta|k)$, which incorporates the adaptive, self-tuning, or system identifying nature of the estimator.

- Adaptive estimation in this context constitutes a joint estimation and multi-hypothesis detection problem [23]. Estimation takes place with respect to the desired state $x(k)$, whereas detection or hypothesis testing occurs with respect to the decision of each model. This inference can be readily seen by noting that $L(k|k;\Theta)$ may be viewed as the likelihood ratio for the following set of pattern recognition problems (one for each element of the vector Θ):

$$H_1 \rightarrow z(k) = H(k;\Theta)x(k;\Theta) + v(k) \tag{50}$$

$$H_0 \rightarrow z(k) = v(k) \tag{51}$$

- Each value of the above detection problems tests the hypothesis that the observed data was generated by the model indexed by the parameter value Θ against the null hypothesis that the data was generated by noise only. This global and unifying viewpoint of estimation, identification, and detection as different manifestations of the same problem has been proposed and extensively studied by Lainiotis [17], [20], [23]. Estimation and identification, in the Lainiotis framework, can be formulated as a detection problem, and, in turn, detection and pattern recognition may be considered as estimation and system identification formulations [26]–[28].

- The partitioning theorem also provides the exact error covariance matrix expression, equation (45), in integral partitioned form, which is realizable with a minimum of additional computations since the quantities involved are already available from the evaluation of the adaptive estimator. As such, it is useful for on-line monitoring of the estimator performance. It is noteworthy to state that the exact error covariance expression is the only exact and explicit one obtained in nonlinear or adaptive scenarios.

- The situation outlined above and described in equations (44)–(49) pertains to the case where the probability density function associated with Θ is a continuous function of Θ. Under such operation, however, one is faced with the need for a non-denumerable infinity of parallel linear filters for the exact realization of the optimal estimator. The usual approximation performed to overcome this difficulty is to represent the probability density of Θ with a finite sum, i.e. to discretize the sample space of Θ. There exist, of course, cases in which the sample space is in itself naturally discrete. In those cases, the

integrals in equations (44)–(49) are replaced by summations running over all possible values of the vector parameter Θ. It is comforting to know that when the true parameter value lies inside the sample space, the adaptive estimator converges to this value. When the true parameter value is not included in the assumed sample space, the estimator converges to that value in the sample space that is 'closest' to the true value in the sense of Kullback's information measure minimization [13].

- From a practical standpoint, the partitioning formulae yield realizations of the optimal or suboptimal estimators that are computationally attractive, numerically robust with respect to failure of any of the parallel processing units, and whose implementation may be accomplished in a pipeline or parallel processing mode [5], [15], [27], [29].

E. Partitioned Adaptive Filters for Nonlinear Models

In this section, the partitioning filters proposed by Lainiotis [21], [24], [26], [27] are presented for the basic nonlinear model given by the following equations:

$$x(k + 1) = f(x(k),k;\Theta) + \Gamma(k;\Theta)w(k), \tag{52}$$

$$z(k) = h(x(k),k;\Theta) + v(k), \tag{53}$$

where $x(k)$ is the state vector of the system, $f(x(k), k;\Theta)$ is a (possibly unknown) nonlinear function of the state, $\Gamma(k;\Theta)$ is the (possibly unknown) standard deviation of the noise term $w(k)$, $z(k)$ is the measurement vector, $h(x(k), k;\Theta)$ defines a nonlinear observation matrix of the state that may contain uncertainties, and $v(k)$ is the additive noise that corrupts the measurements. The unknown parameters are denoted by the vector Θ, which, if known, would completely specify the model. Moreover, Θ is considered to be a random variable with known or assumed a-priori density $p(\Theta|0) = p(\Theta)$. The processes $w(k)$ and $v(k)$ are still uncorrelated when conditioned on Θ, with covariances $Q(k;\Theta)$ and $R(k;\Theta)$, respectively. The initial state vector $x(0)$ is conditionally Gaussian for given Θ, with mean $\hat{x}(0|0;\Theta)$ and covariance $p(0|0;\Theta)$, and is conditionally uncorrelated with the sequences $w(k)$ and $v(k)$.

To seek an optimal solution to the above problem is hopeless. The partitioning approach can produce an efficient solution for the estimate $\hat{x}(k|k)$, given that the conditional estimates $\hat{x}(k|k;\Theta)$, which are the approximate estimates matched to a specific value of the vector parameter Θ, are available through normal estimation techniques. As explained in earlier discussions, nonlinear estimators require infinite-dimensional processes and cannot be, in general, implemented. Since approximate structures can be formulated, though, as in the case of the EKF, the partitioned algorithm, at least intuitively, may be realizable. For example, one may construct a bank of EKFs in parallel, each one matched to an appropriate value Θ_i such that the overall vector $\Theta = [\Theta_1,...\Theta_i,...\Theta_m]^T$ spans the space of the unknown parameters. Then the partitioned estimator is used to select the EKF conditional model matched to the correct value of the unknown parameter (or the one that is closest to it). The design is herein referred to as the adaptive Lainiotis extended filter (ALEF). What one has to remember is that the estimates $\hat{x}(k|k; \Theta)$ in the individual filters

of the parallel configuration, will only be approximations of the conditional means, and this is also true for the corresponding error covariances.

The integrals of the formulation will eventually be substituted by a finite sum of discrete values for digital computer implementation. In other words, a concatenation of two approximations is put into practice, namely, one for the nonlinear estimator of the individual subparts in the bank of filters, and another to avoid the infinite amount of parallel structures that is needed to cover the exact sample space of the unknown parameter Θ. As a result, there is no theoretical justification or guidelines for how well the nonlinear partitioned algorithm can perform or even if it manages to converge. Monte Carlo techniques should be used to verify adequate performance manifestation.

5. NONLINEAR FILTERS FOR LIDAR UNCERTAIN SYSTEMS

A. ALEF Conditional Filters

We now redirect our attention to the problem of signal processing of LIDAR log power returns. To design a nonlinear estimator for an unknown LIDAR model such as the one of Section II, we define a vector Θ that contains the model uncertainties as follows:

$$\Theta = \begin{bmatrix} \Theta_z \\ \Theta_w \end{bmatrix}, \tag{54}$$

The EKF equations are modified to include the vector Θ as follows: State Estimate Propagation:-

$$\hat{x} (k + 1|k; \Theta) = \hat{x} (k|k; \Theta), \tag{55}$$

without multiplicative noise, or

$$\hat{x} (k + 1|k; \Theta) = \begin{bmatrix} \hat{x}_1 (k|k; \Theta) \\ 1 \end{bmatrix}, \tag{56}$$

when one considers measurements with speckle.

Error Covariance Propagation:-

$$P (k + 1|k; \Theta) = F (k) P (k|k; \Theta) F^T (k) + Q (k; \Theta). \tag{57}$$

Filter Gain:-

$$K (k + 1; \Theta) = P (k + 1|k; \Theta) H (k + 1; \Theta) [H (k + 1; \Theta) P$$
$$(k + 1|k; \Theta) H^T (k; \Theta) R (k)]^{-1}. \tag{58}$$

State Estimate Update:-

$$\hat{x}(k + 1|k + 1; \Theta) = \hat{x}(k + 1|k; \Theta) + K(k + 1; \Theta)[Z(k + 1) - h(\hat{x}(k + 1|k; \Theta),$$

$$k + 1; \Theta)]. \tag{59}$$

Error Covariance Update:-

$$P(k + 1|k + 1; \Theta) = [I - K(k + 1; \Theta) H(k + 1; \Theta)] P(k + 1|k; \Theta). \tag{60}$$

The above equations take different form depending upon the specific problem at hand and the approximation one wishes to include. For the LIDAR system, we consider here two cases. If the problem is treated without multiplicative noise, then the system equations become simply scalar since only one state requires estimation. In that scenario, we can define the following (based on the discussion in section II):

$$F(k) = 1, \tag{61}$$

$$H(k + 1; \Theta) = \Theta_Z exp(x_1(k + 1; \Theta)), \tag{62}$$

$$h(x(k + 1; \Theta)) = \Theta_Z exp(x_1(k + 1; \Theta)). \tag{63}$$

The next level of complexity introduces an extra state to account for the speckle returns to be estimated. The problem now becomes two-dimensional, with the first state representing the signal as previously and the second the speckle term. For measurements with speckle, the following definitions apply as they can be used in the EKF algorithm:

$$F(k) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \tag{64}$$

$$H(k + 1; \Theta) = [\Theta_Z x_2(k + 1; \Theta) exp(x_1(k + 1; \Theta)) \Theta_Z exp(x_1(k + 1; \Theta))]]. \tag{65}$$

$$h(x(k + 1; \Theta)) = \Theta_Z x_2(k + 1; \Theta) exp(x_1(k + 1; \Theta)). \tag{66}$$

With the above definitions, the EKF and ALEF algorithms can now be readily applied.

B. Augmented EKF

Before we leave the discussion on identification filters, we draw some attention to the use of an adaptive scheme as an alternative to the partitioning approach. Although sometimes adequate, the adaptation to the real parameter values is often slow and it is no match to the partitioning theory reviewed earlier. A direct comparison between the two will be used in a subsequent section for identification of the uncertainties existing in the measurement

equation. It is noted that for unknown structures that contain state dependent noise terms, the technique is not applicable and more elaborate schemes have to be used.

Let as assume a vector of unknown parameters can be defined as follows:

$$\Theta = \begin{bmatrix} \Theta_1 \\ \cdots \\ \Theta_N \end{bmatrix}.$$ (67)

Then the actual state problem of equation (52) can be augmented to accommodate the unknown vector as follows:

$$x_a(k+1) = [x_a(k+1)\ \Theta(k+1)]^T,$$ (68)

$$x_a(k+1) = \begin{bmatrix} f(x(k)) \\ \Theta(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(k).$$ (69)

Equations (68) and (69) can be used with equation (53) of the measurements to jointly estimate the unknown parameters of the vector Θ, which is treated as a static noiseless state, and the actual states of interest, which remain as before. As mentioned before, the approach suffers when the unknown parameter is a state dependent noise term since the augmentation is not clear to perform.

6. PERFORMANCE EVALUATION

We now present an overview of the results obtained in the current study. We assume that the unknown parameters occur as discussed in section II, that is, as an uncertainty that is noise dependent (and the EKF is simply mismatched) and as an uncertainty that appears in the measurements (where the EKF can be augmented and made adaptive). In either case, the EKF approach was compared vis a vis the partitioning theory based methods. We consider first the case of the unknown noise covariance for both the speckle and no speckle schemes, resulting to a scalar and a two-dimensional state space representation, respectively. Then we investigate the uncertainty that exists in the measurements, again for both the cases of speckle present or speckle absent.

A. Log Power Estimation Without Speckle: Unknown Noise Covariance $Q_1(k)$

In this example, the problem considered is the estimation of the log power given measurements contaminated by additive noise but without multiplicative noise; thus the speckle is neglected and the example has the application of a direct detection system.

Signal sequences, containing 200 data points were generated using equation (5), and observation series using equation (2). In equation (2) we set $\Theta_Z = 1$. The signal noise covariance Q_1 is the only unknown, assumed to be varying between 0.0005 and 0.05 uniformly. The uniform distribution corresponds to the worst case scenario to demonstrate
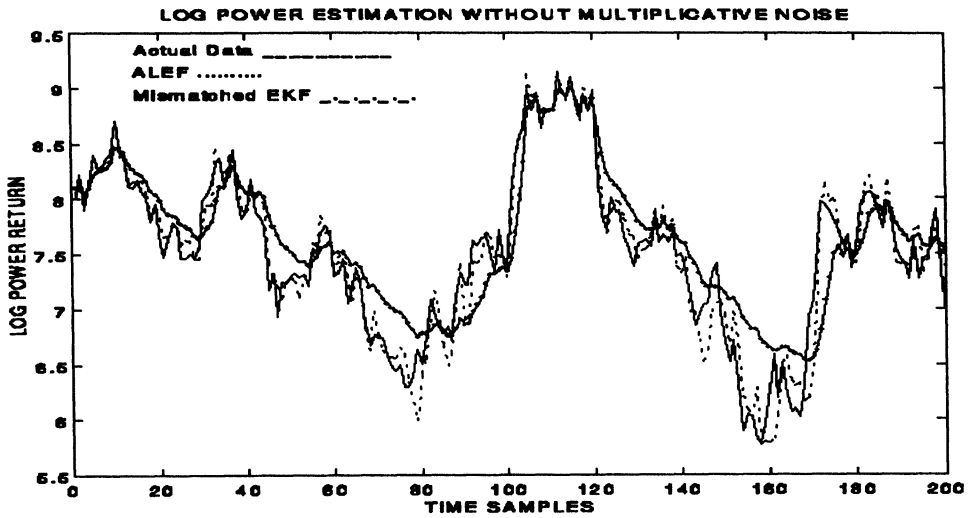
Figure 1 Simulated Log Power with Mismatched EKF and ALEF Estimates (R = 300). Unknown Q.

no a-priori knowledge of the unknown parameter's probability distribution. The space span for the noise variance Q_1 was chosen based on values from real measurements of LIDAR returns found in [42]. Usual EKF techniques cannot be applied here since the model is not completely known. The need for an adaptive filter is apparent.

The EKF was used as a simple nonlinear estimator without any inherent mechanism for estimation of the unknown parameters. In other words, the filter does not know the exact dynamics of the model because the signal noise variance has a uniform distribution. Furthermore, since the unknown is a state dependent noise term, the simple adaptive EKF scheme examined in the previous section cannot be used. The recursive algorithm starts with an initial estimate, $\hat{x}(0|0) = 8.1$, and initial error covariance, $P(0|0) = 0.09$. The ALEF is designed with three EKFs in parallel, each one matched to a specific value of Q_1 to cover the entire range of the assumed distribution of Q_1. The approach becomes a multi-model estimator that simultaneously tries to track the state trajectories as well as the unknown parameters. The first model was designed with $Q_1 = 0.05$, the second with $Q_1 = 0.001$, and the third with $Q_1 = 0.0005$. The estimates given by the EKF and ALEF are shown in Figure 1 along with the true state of the system. It is obvious that the ALEF estimator tracks the true trajectory much closer than the mismatched EKF. In order to assess the performance of the above filters, the mean square error, averaged over 50 Monte-Carlo runs, was used and the results for both filters are depicted in Figure 2. The improvement in the error performance is found significant.

B. Log Power Estimation With Speckle: Unknown Noise Covariance $Q_1(k)$

The simulation discussed above is extended to include the multiplicative noise so that the signal measurements are generated using equation (7) with $\Theta_Z = 1$. To simulate lower order speckle, $x_2(k)$ is generated assuming chi-square statistics of order 14 (see [40]). The speckle is decorrelated between successive measurements and this justifies the approxima-
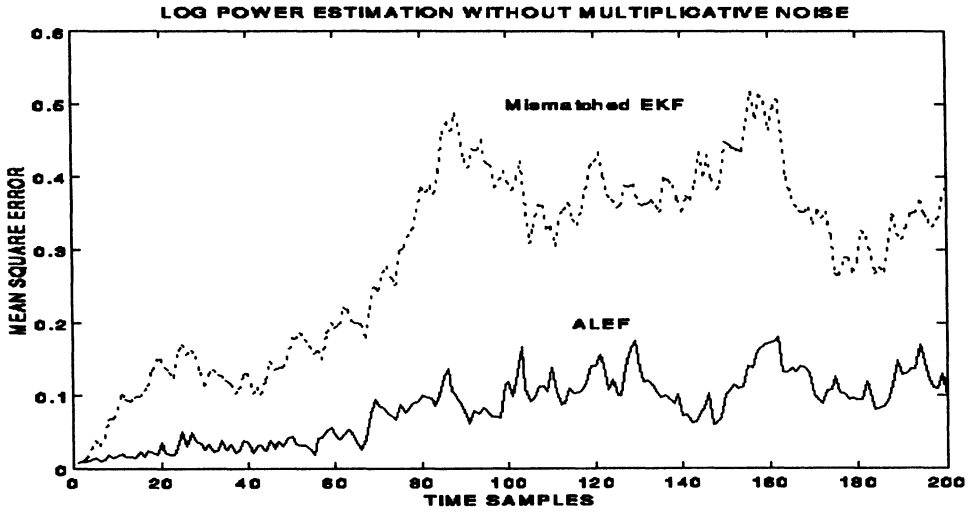
Figure 2   Mean Square Error of the Mismatched EKF and ALEF. Unknown Q_1.

tion given in equation (6). All the other terms are similar to those of the previous example and the results for this example are given in Figures 3, 4, and 5. The ALEF multi-model estimator shows a successful detection of the model that most closely relates to the actual system. In Figure 6, the histogram (30 bins) of the speckle power estimates is shown to verify the chi-square statistics used in the data generation. The frequency distribution of the filter estimates shows the expected probability density function for the low-order speckle.
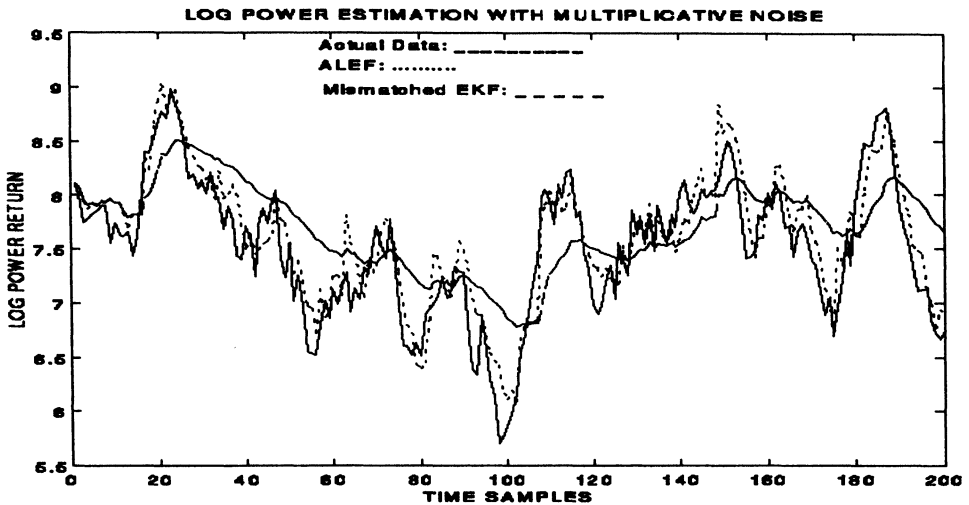
Figure 3   Simulated Log Power with Mismatched EKF and ALEF Estimates (R = 300). Unknown Q.

LOG POWER ESTIMATION WITH MULTIPLICATIVE NOISE
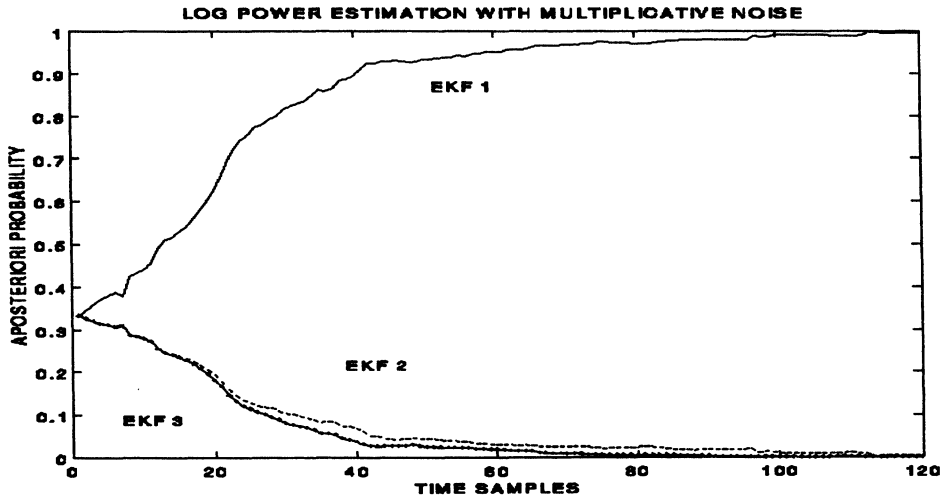
Figure 4   A-posteriori Probability Values for the ALEF Blocks. Unknown Q_1.

C. Log Power Estimation Without Speckle: Unknown Matrix H(k)

Here we shift our attention to the unmodeled measurement equation, assuming that the exact structure of the matrix $H(k)$ in the linearized equation (62), which we denote by Θ_z, is unknown. The unknown parameter is assumed to be taking any of the discrete values 0.1, 1, and 10, to model a possible attenuation/amplification that may occur as the measurements arrive at the receiver. The noise covariance $Q_1(k)$ is now set to 0.001 and held constant.

LOG POWER ESTIMATION WITH MULTIPLICATIVE NOISE
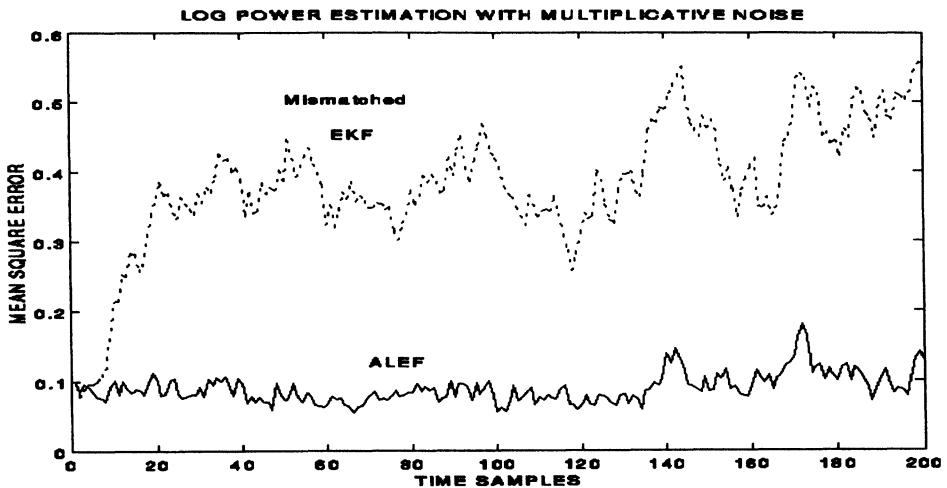
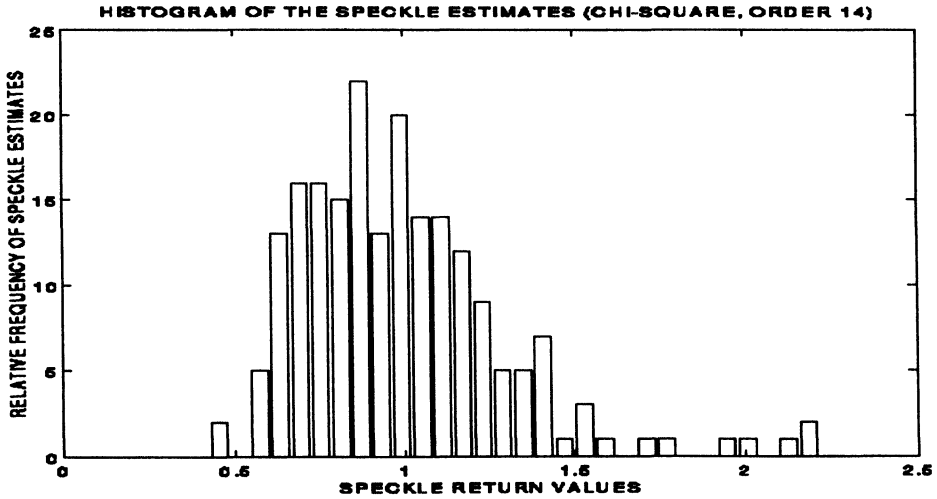Figure 5   Mean Square Error of the Mismatched EKF and ALEF. Unknown Q_1.

Figure 6 Histogram of the Speckle Estimates to Verify Chi-Square Statistics of Order 14 Used in the Data Generation. Unknown Q_1.

Signal sequences, containing 200 data points were once more generated using equation (5), and observation series using equation (2). Usual adaptive EKF techniques can be applied here since the unknown is not a state dependent noise term. One can augment the system to include the unknown parameter as an additional state. Then the filter will adaptively try to converge to the real value of the unknown parameter. The initial value for the state to identify the parameter was set to unity, the middle point of the three discrete values that the parameter can assume in the actual generation of the data.
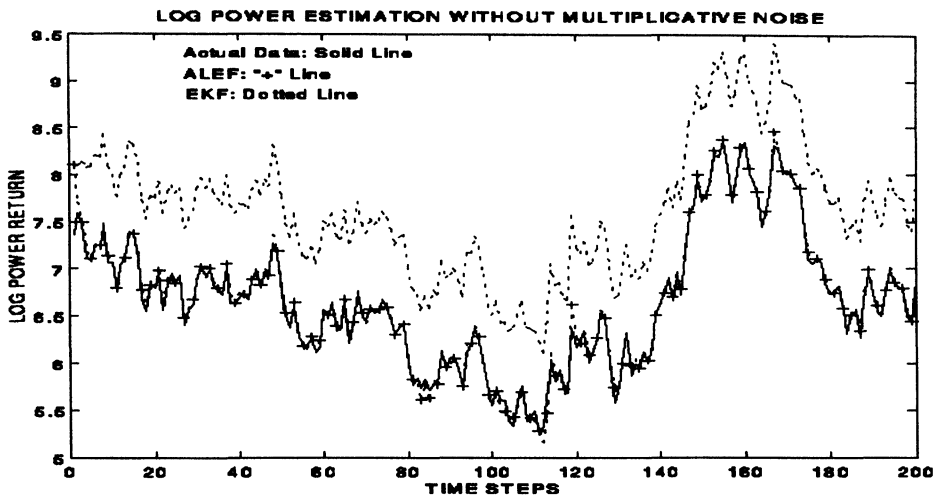

Figure 7 Simulated Log Power with Adaptive EKF and ALEF Estimates. Unknown H.

LOG POWER ESTIMATION WITHOUT MULTIPLICATIVE NOISE
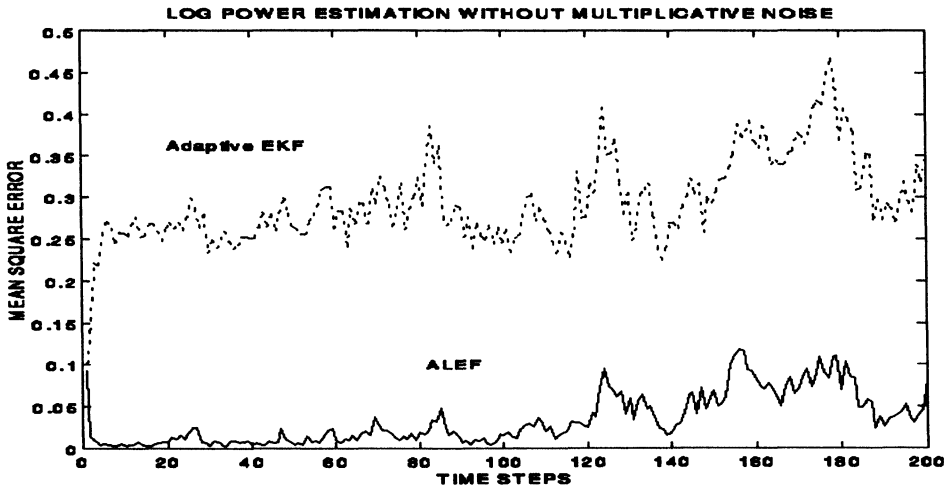

Figure 8   Mean Square Error of the Adaptive EKF and ALEF. Unknown H.

The recursive algorithm starts with an initial estimate, $\hat{x}(0|0) = 8.1$, and initial error covariance, $P(0|0) = 0.09$. The ALEF is designed with three EKFs in parallel, each one matched to a specific value of Θ_z to cover the entire range of its assumed distribution. The estimates given by the EKF and ALEF are shown in Figure 7 along with the true state of the system. The performance of the above filters is assessed by their mean square error averaged over 50 Monte-Carlo runs, and the results for both filters are depicted in Figure 8. The filtering capacity of the ALEF estimator is shown to be much superior than the adaptive EKF. The reason is the failure of the latter to converge to the correct value of the unknown parameter, which results in the introduction of significant bias.
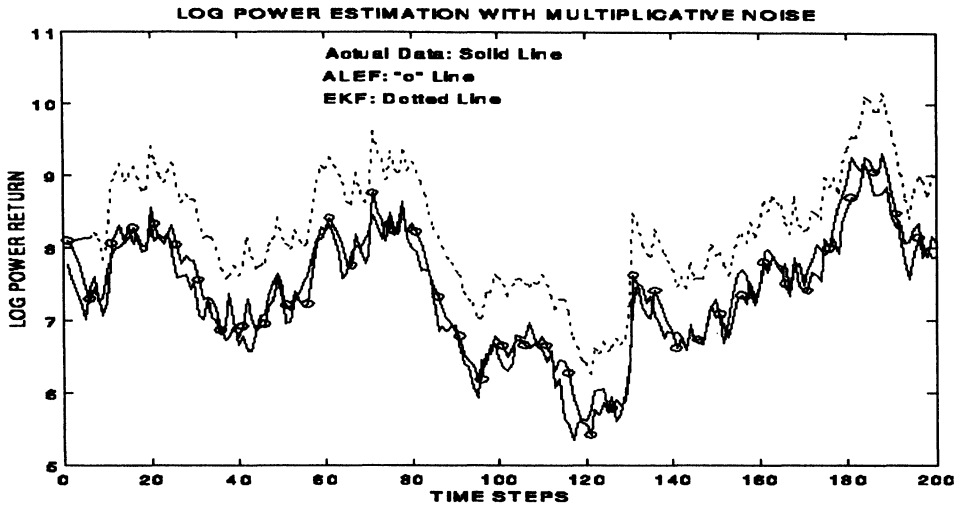
LOG POWER ESTIMATION WITH MULTIPLICATIVE NOISE

Figure 9   Simulated Log Power with Adaptive EKF and ALEF Estimates. Unknown H.
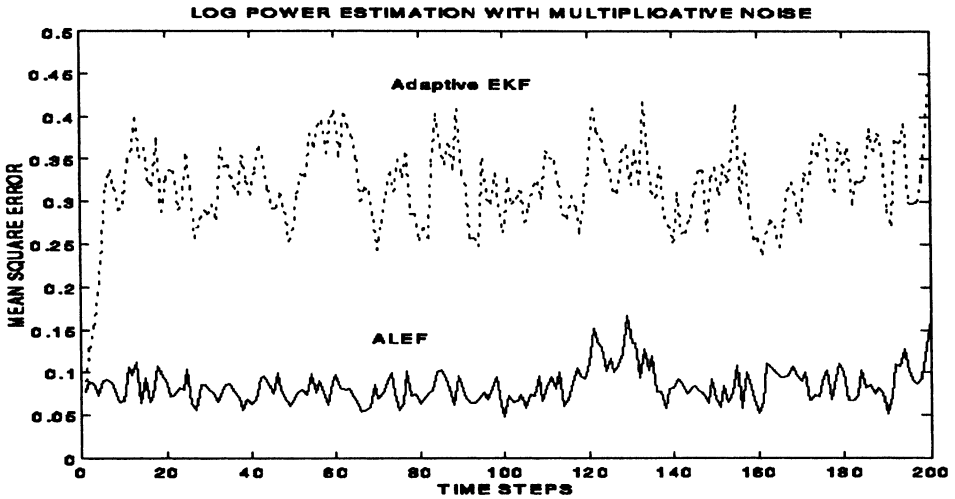
Figure 10 Mean Square Error of the Adaptive EKF and ALEF. Unknown H.

D. Log Power Estimation With Speckle: Unknown Matrix H(k)

The last simulation to be discussed is extended to include the multiplicative noise so that the signal measurements are generated using equation (7) with fixed Q_1 $(k) = 0.001$ in equation (5). To simulate speckle in equation (6), x_2 (k) is again generated assuming chi-square statistics of order 14. All the other terms are similar to those of the previous example.

The results for this example are given in Figures 9 and 10. The comparison is made between the augmented EKF and ALEF. We call the augmented EKF adaptive since it is now capable of adjusting to the unknown parameter and is no longer mismatched. The ALEF is significantly superior to the adaptive EKF as demonstrated by our findings in both log power estimation and error performance.

E. Discussion

We have shown that the partitioning approach as a means of model selection when uncertain environments exist can perform extremely well and much better than conventional means of estimation for the simple cases of independently unknown time invariant parameters. The performance improvement over mismatched or slowly adaptive schemes is expected to extrapolate itself for more realistic scenarios where the unknowns can be time varying. The degree of approximation that is needed is dependent upon the time and computational requirements to efficiently implement the parallel structures of the partitioning theory.

7. CONCLUSION

The adaptive Lainiotis extended filter (ALEF) has been shown to be an efficient estimator for LIDAR applications, and, in particular, far more superior than the widely used

Table 1 Error Comparison for the Simulation Cases Studies

Simulation Examined	EKF MSE	ALEF MSE	Percent Improvement
No Speckle, Unknown Q_1	0.41	0.14	192%
Speckle, Unknown Q_1	0.38	0.09	322%
No Speckle, Unknown H	0.29	0.03	866%
Speckle, Unknown H	0.32	0.08	300%

extended Kalman filter (EKF). Through extensive computer simulation studies, it has been established that the ALEF can offer up to 300% improvement over the conventional EKF for LIDAR problems involving multiplicative noise or speckle.

Unknown parameters play an important role in the overall estimator performance. The simulations show that the EKF develops a significant bias error from imperfect knowledge of the signal noise covariance. The ALEF estimator adjusts to changes in the noise within a few time steps and eliminates the significant bias error developed by the mismatched EKF. When an adaptive scheme was used for the EKF, by augmenting the states to accommodate the unknown parameters, the conclusion remained the same. The ALEF multi-model estimator still outperforms its single model augmented counterpart, primarily because of the slow adaptation response the latter exhibits. Due to the highly decoupled structure of the Lainiotis approach, the time required to realize the filter remains essentially the same as with the simple EKF. Important results of this work are summarized in Table 1.

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Elbert E. Neher Macau, Laboratório Associado de Matemática Aplicada e Computação (LAC), Instituto Nacional de Pesquisas Espaciais (INPE), São Josè dos Campos, 12227-010 São Paulo, Brazil ; elbert@lac.inpe.br

Celso Grebogi, Department of Physics, King's College, University of Aberdeen, Aberdeen AB24 3UE, UK; grebogi@abdn.ac.uk