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Research Article

Imperfect Reworking Process Consideration in Integrated Inventory Model under Permissible Delay in Payments

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This study develops an improved inventory model to help the enterprises to advance their profit increasing and cost reduction in a single vendor single-buyer environment with general demand curve, adjustable production rate, and imperfect reworking process under permissible delay in payments. For advancing practical use in a real world, we are concerned with the following strategy determining, which includes the buyer's optimal selling price, order quantity, and the number of shipments per production run from the vendor to the buyer. An algorithm and numerical analysis are used to illustrate the solution procedure.

1. Introduction

In this highly competitive globalized environment, enterprises are forced to pace their supply according to the requirements of customers. Their initiative to have quick customer response will help them to occupy the market and become the market leaders. Many enterprises attempt to manage their supply chain effectively. One useful technique to achieve this target is to use just-in-time (JIT) and the key to a successful JIT system is to be able to benefit both the vendor and buyer. This is done through the mutual negotiations and agreements on how the savings are divided (Hahn et al. [1]). JIT systems in today's supply chain environment require the creation of a new spirit of cooperation between the buyer and the vendor to gain and maintain a competitive advantage. As Ha and Kim [2] have pointed out, the integrated inventory model can contribute significantly to this vendor-buyer relationship. In Ohta and Furutani's [3] model, a supply chain system, which consists of the supplier, the buyer, and the customer, where the buyer corresponds to a wholesaler, analyzes the effect of customer order cancellations on (s, S) inventory policies for the supplier and the buyer in the supply chain system.

The spirit of cooperation among enterprises is needed to improve the effectiveness of the supply chain. One of the common strategies in the business cooperation is that the buyers are offered a permissible delay period to pay back for the goods bought without paying any interest. During this period, the buyer does not need to pay interest on goods kept in stock. However, higher interest is charged if the payment for the goods is not paid by the end of this period. For the vendor, he has the benefit of attracting the buyer to purchase his goods in large batches. Therefore, the existence of the permissible delay period will promote a vendor's selling and reduce on-hand stock level. Simultaneously, a buyer can earn the interest of the sales revenue and reduce the holding stock because of the reduced amount of capital invested in stock for the duration of the permissible delay period. Goyal and Cárdenas-Barrón [4] first developed an EOQ model with constant demand rate under conditions of permissible delay in payments. He supposed that no deterioration occurs and the capacity of the warehouse is unlimited. Besides, he also disregarded the difference between the selling price and the purchase cost, and concluded that the economic replenishment time interval and order quantity usually increase marginally under permissible delay in payments. Aggarwal and Jaggi [5] extended the Goyal's model to deteriorating items. Jamal et al. [6] farther extended the model of Goyal [7] to permit shortage and deterioration. Yang and Wee [8] developed a single-vendor, multibuyers inventory policy of a deteriorating item with a constant demand rate. Recently, Teng [9] amended the Goyal's model by considering the difference between the selling price and the purchase cost, and found an alternative conclusion. Abad and Jaggi [10] provided an integrated approach to the vendor for determining his pricing and credit policy when end demand is price sensitive. They considered the vendor-buyer relationship under a noncooperative as well as a cooperative situation and supposed that the vendor follows a lot-for-lot shipment policy. Huang and Yao [11] aimed at optimally coordinating inventory for a deteriorating item among all the partners in a supply chain system with a single vendor and multiple buyers so as to minimize the average total costs. Teng et al. [12] then improved Teng [9] by supposing that demand rate is price sensitive. Ouyang et al. [13] proposed a model with adjustable production rate under the condition of permissible delay in payments. Huang et al. [14] want to extend that fully permissible delay in payments to the supplier would offer the retailer partially permissible delay in payments. The retailer must make a partial payment to the supplier when the order is received. Then the retailer must pay off the remaining balance at the end of the permissible delay period. Their research showed that the trade credit strategy, as permissible delay in payments, could be a win-win strategy. Their analysis also identified that the total channel profit would increase while the vendor and the buyer could cooperate to share necessary business information with each other and balance the rate between production and market demand.

It is impractical that the above integrated vendor-buyer inventory models are assumed that the produced or received products are perfect without any imperfect quality item. In fact, due to the deteriorating production process of the vendor and the damage during the transportation process from the vendor to the buyer, an arrival order batch for the buyer may contain some percentage defectives. Therefore, the conventional integrated inventory model without quality consideration is inappropriate for the situation in which an arrival batch contains some imperfect quality items. Porteus [15] first incorporated the effect of defective items into the classical EOQ model and introduced the alternative of investing in process quality improvement through reducing uncontrollable process quality parameters. Rosenblatt and Lee [16] also considered the effect of an unreliable production process into the EPQ model.

Their results showed that the average percentage of defective items would be increased by reducing the lot size. Lee and Rosenblatt [17] added process inspection consideration into production runs so that the change, which could move to the process out of control, could be inspected and restored earlier than classical EOQ models. Schwaller [18] extended the EOQ model by joining a known defective rate assumption into the incoming batches and that fixed and variable screening costs are incurred in finding and expelling. Zhang and Gerchak [19] considered a joint lot sizing and inspection policy in an EOQ model where a percentage defective is random. Cheng [20] recommended an EOQ model with demand-dependent unit production cost and imperfect production processes. He formulated the problem as a geometric programing and solved it to get closed-form optimal solutions. Recently, Ben-Daya and Hariga [21] examined the effect of defective items on production scheduling and established a mathematical model to illustrate the scheduling questions. Salameh and Jaber [22] considered a joint lot sizing and inspection policy under an EOQ model for items with imperfect quality. Their results showed that economic lot size quantity tends to increase as the average percentage of imperfect quality items increase. This contradicts with the finding of Rosenblatt and Lee [16]. They also considered that poor-quality items should be sold as a single batch at a discounted price prior to receiving the next shipment. Hayek and Salameh [23] studied an inventory operating policy under the condition that imperfect quality items would be reworked where shortages are allowed and backordered. Goyal [7] proposed a simple approach to determine the economic production quantity for items with imperfect quality.

From the above-mentioned arguments, for advancing practical use in a real world, this paper develops an integrated inventory model with process unreliability consideration and permissible delay in payments. Imperfect quality items are handled in the same way as proposed in Salameh and Jaber [22]. Yu et al. [24] developed a production-inventory model considering a deteriorating item with imperfect quality and partial backordering. This paper further extends the model of Ouyang et al. [13] to imperfect quality items. The main purpose is to maximize the joint total profit from the perspective of both the vendor and the buyer with the following strategy determining, which includes the buyer's optimal selling price, order quantity, and the number of shipments per production run.

The rest of this paper is organized as follows. The following section describes the notations and assumptions made herein. Section 3 reports on the proposed mathematical model and Section 4 establishes the solution procedure. Section 5 provides numerical examples to illustrate the analysis of Sections 3 and 4. The final section draws the research conclusions.

2. Notations and assumptions

To establish the proposed model, the following notations are used.

Notations

$D(p)$: Average demand per year, as a function of the selling price P

A: Vendor's production rate, $A > D$

Q: Buyer's order quantity per order

c_V: The unit production cost for the vendor

c_B: The unit purchasing cost for the buyer

P: The unit selling price for the buyer, a decision variable

S_V: Setup cost per production run for the vendor
S_B: Ordering cost per order for the buyer
h_V: The unit holding cost rate for the vendor excluding interest charges
h_B: The unit holding cost rate for the buyer excluding interest charges
F: Transportation cost per shipment
n: The total number of shipments per production run from the vendor to the buyer, a positive integer and a decision variable
L: Buyer's replenishment time interval between successive deliveries and a decision variable
m: Buyer's permissible delay period offered by the vendor per order
I_{Vp}: Vendor's capital opportunity cost per dollar per year
I_{B_p}: Buyer's capital opportunity cost per dollar per year
I_{Be}: Buyer's interest earned per dollar per year
Z: Percentage of defective items in Q, a random variable
$f(z)$: Probability density function of z
ω: The unit inspecting cost
c_R: Repair cost per item of imperfect quality for the vendor
$\text{TP}_V(n, p, Q)$: The vendor's total annual profit
$\text{TP}_B(p, Q)$: The buyer's total annual profit
$\text{JTP}(n, p, Q)$: The joint total annual profit
$\text{ETP}_V(n, p, Q)$: The expected vendor's total annual profit with Z
$\text{EJTP}(n, p, Q)$: The expected joint total annual profit with Z.

The assumptions made in the paper are as follows.

Assumptions

(1) There is a single vendor and single buyer for a single product.

(2) The isoelastic curve the most conventionally assumed is selected as a price-demand function form throughout this model and we set $D(p) = \gamma p^{-\beta}$, where $\gamma > 0$ is a scaling factor, and $\beta \geq 1$ is an index of price elasticity.

(3) The production rate, A, is adjustable. The ratio between the production rate A and the demand rate D was set to be $A/D = \lambda$, where $\lambda > 1$ is a constant and $A = \lambda D$.

(4) The buyer orders a quantity of Q for each order with an ordering cost S_B; the vendor manufactures at rate A, in batches of size nQ with a lot setup cost S_V; each batch is delivered to the buyer in n equally sized shipments. For each shipment, the buyer brings a transportation cost F.

(5) Successive shipments are scheduled so that the next one arrives at the buyer when his stock from previous shipment has just been consumed.

(6) Shortages are not allowed.

(7) The relationship between the buyer's selling price p, buyer's purchasing cost c_B and vendor's production cost c_V is $p \geq c_B \geq c_V$.

(8) The vendor offers the buyer a permissible delay period m. During this permissible delay period, the buyer sells the items and uses the sales revenue to earn interest at a rate of I_{Be}. At the end of this time period, the buyer pays the purchasing cost to the vendor and the items still in stock bring a capital opportunity cost at a rate of I_{Bp}.

(9) During the vendor's production process, the produced items are continuously reviewed.

(10) Under JIT manufacturing concept, defective items are not allowed. For maintaining JIT spirit and conforming to the truth, we assume that the reworking of defective items starts instantly they are fund in the same batch cycle and these reworked items are of perfect quality.

(11) In a single batch at the end of the vendor's 100% inspecting process, if imperfect quality items are found and the repair cost must be paid.

(12) The time horizon is infinite.

3. Model formulation

In this section, we formulate the model for the reality assuming that the vendor offers the permissible delay in payments to the buyer and imperfect quality items can be produced during a production run. We make use of the imperfect quality items consideration to extend the integrated inventory model established by Ouyang et al. [13]. Imperfect quality items are occurred in the vendor's production process, and these items can be reworked immediately in the same batch cycle. These imperfect quality items being reworked are of perfect quality. So the vendor delivers an order quantity of Q with perfect quality to the buyer and the buyer accepts it over n times.

Figure 1 depicts the behavior of inventory levels for both the vendor and the buyer, which is along the notations and the assumptions shown above. The joint total annual profit for the vendor and the buyer consists of (3.1) the vendor's total annual profit, and (3.2) the buyer's total annual profit.

3.1. The vendor's total annual profit

In each production run, the vendor produces the item in the quantity of nQ with the rate of A and brings a setup cost S_V as the buyer places an order of quantity Q over n times. But, the vendor's manufacturing will produce some imperfect quality items. It is assumed that each batch of size Q produced contains percentage defectives ZQ. From assumption 10 stated above, the quantity of ZQ with imperfect quality must be reworked instantly when they are fund. These reworked items are excellent in quality. So the vendor's production quantity can be divided into two parts: the quantity of $n(1-Z)Q$ with perfect quality and the reworked quantity of nZQ with perfect quality. Therefore, the total production quantity for the vendor is still nQ and the buyer would receive it in n batches, which each has a quantity of Q with no defect. Following the above notations and assumptions, the components in the vendor's total annual profit function are

(i) sales revenue per year = $D(c_B - c_V)$,

(ii) setup cost per year = $S_V/nL = S_V D/nQ$,

(iii) inventory holding cost including financing cost per year = $(c_V h_V + c_V I_{Vp}) \times (Q/2)[n(1-1/\lambda) - 1 + 2/\lambda]$,

(iv) opportunity cost per year for offering the permissible delay period $m = c_B I_{Vp} \times Dm$,

(v) inspecting cost per year = $c_R \times nZQ/nL = c_R ZD$.

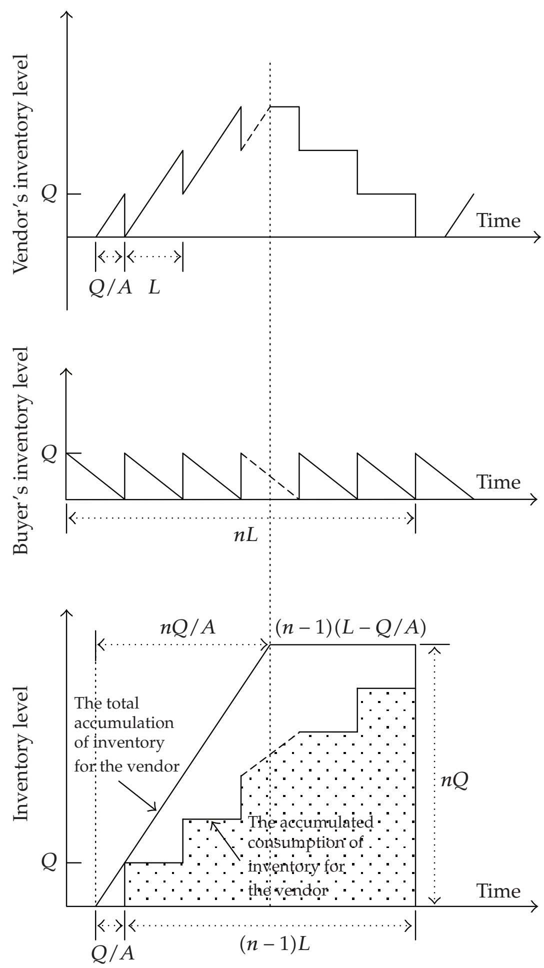

Figure 1: An integrated inventory system for the vendor and the buyer.

Thus, the vendor's total annual profit, $\mathrm{TP}_V(n,p,Q)$, can be shown to be

$$\begin{aligned}
&\mathrm{TP}_V(n,p,Q)\\
&\quad= \text{sales revenue} - \text{setup cost} - \text{inspecting cost} - \text{holding cost} - \text{opportunity cost}\\
&\quad= D(c_B - c_V) - \frac{S_V D}{nQ} - D\omega - c_R Z D - \frac{c_V Q(h_V + I_{Vp})}{2}\left[n\left(1-\frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right] - c_B I_{Vp} D m.
\end{aligned} \tag{3.1}$$

3.2. The buyer's total annual profit

In this model, the buyer's replenishment time interval between successive deliveries is $L = Q/D$. The buyer brings an ordering cost S_B and a transportation cost F for each order of quantity Q. The buyer's total annual profit consists of the following components:

(i) sales revenue per year = $D(p - c_B)$,

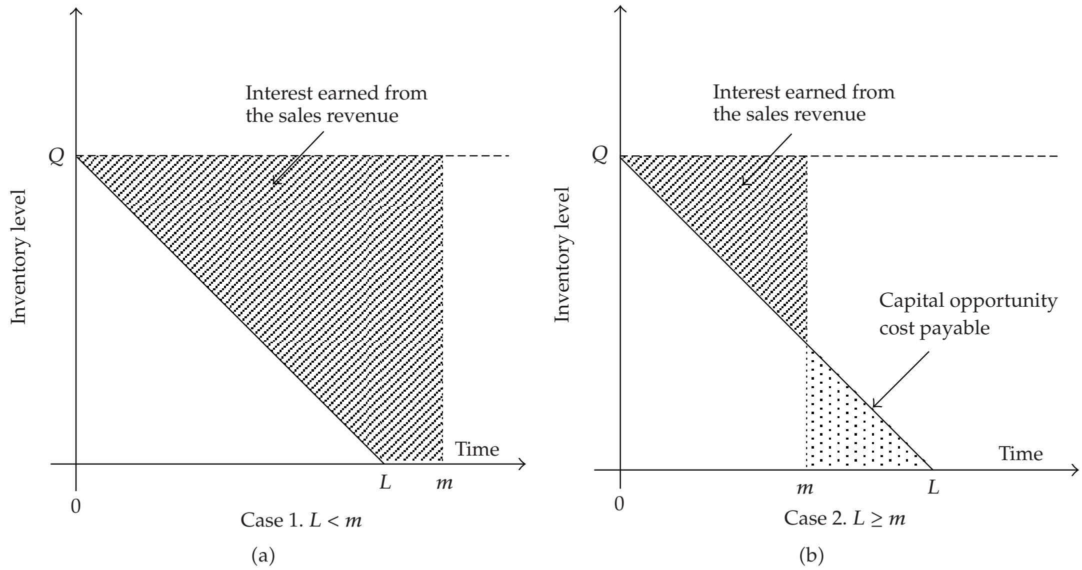

Figure 2: Inventory systems for the buyer based on the relation between L and m.

(ii) cost of placing orders per year = $S_B/L = S_B D/Q$,

(iii) transportation cost per year = $F/L = FD/Q$,

(iv) inventory holding cost per year = $c_B h_B \times Q/2$,

(v) interest earned from the sales revenue received during the permissible delay period m,

(vi) capital opportunity cost payable for the items unsold after the permissible delay period m (notice that this cost only exists if $L \geq m$).

Considering the components (v) interest earned, and (vi) capital opportunity cost, the model has the following two possible cases based on the values of L and m. These two cases are depicted graphically in Figure 2.

Case 1 ($L < m$). We first consider Case 1 in Figure 2, where $L < m$, the component (v), the interest earned per year at a rate of I_{Be} in the time span $[0, m]$ is $pI_{Be}(Dm - Q/2)$. In addition, the component (vi), capital opportunity cost payable per year during the time span $[0, m]$ does not exist.

From the above discussions, total profit per year for the buyer, $\mathrm{TP}_{B1}(p, Q)$, is given by

$$\begin{aligned}
&\mathrm{TP}_{B1}(p, Q) \\
&\quad = \text{sales revenue} - \text{ordering cost} - \text{transportation cost} - \text{holding cost} + \text{interest earned} \\
&\quad = D(p - c_B) - \frac{S_B D}{Q} - \frac{FD}{Q} - c_B h_B \frac{Q}{2} + pI_{Be}\left(Dm - \frac{Q}{2}\right).
\end{aligned} \tag{3.2}$$

Case 2 ($L \geq m$). For Case 2 in Figure 2, where $L \geq m$, the component (v), the interest earned per year at a rate of I_{Be} during the time span $[0, m]$ is $pI_{Be}(Dm)^2/2Q$. Next, the component (vi), capital opportunity cost payable per year during the time span $[m, L]$ is $c_B I_{Bp}(Q - Dm)^2/2Q$.

As a result, total profit per year for the buyer, $\mathrm{TP}_{B2}(p, Q)$, is

$$\begin{aligned}\mathrm{TP}_{B2}(p,Q) &= \text{sales revenue} - \text{ordering cost} - \text{transportation cost} - \text{holding cost} \\ &\quad + \text{interest earned} - \text{opportunity cost} \\ &= D(p - c_B) - \frac{S_B D}{Q} - \frac{FD}{Q} - c_B h_B \frac{Q}{2} + \frac{pI_{Be}(Dm)^2}{2Q} - \frac{c_B I_{Bp}(Q - Dm)^2}{2Q}.\end{aligned} \tag{3.3}$$

3.3. *The expected joint total annual profit*

Hence, the joint total annual profit function, $\mathrm{JTP}(n, p, Q)$, can be expressed as

$$\mathrm{JTP}(n,p,Q) = \begin{cases} \mathrm{JTP}_1(n,p,Q) = \mathrm{TP}_V(n,p,Q) + \mathrm{TP}_{B1}(p,Q), & \text{if } L < m \\ \mathrm{JTP}_2(n,p,Q) = \mathrm{TP}_V(n,p,Q) + \mathrm{TP}_{B2}(p,Q), & \text{if } L \geq m, \end{cases} \tag{3.4}$$

where

$$\begin{aligned}\mathrm{JTP}_1(n,p,Q) &= D(p - c_V - \omega - c_R Z) - \frac{D}{Q}\left(\frac{S_V}{n} + S_B + F\right) - \frac{c_V Q(h_V + I_{Vp})}{2}\left[n\left(1 - \frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right] \\ &\quad - c_B I_{Vp} Dm - c_B h_B \frac{Q}{2} + pI_{Be}\left(Dm - \frac{Q}{2}\right), \\ \mathrm{JTP}_2(n,p,Q) &= D(p - c_V - \omega - c_R Z) - \frac{D}{Q}\left(\frac{S_V}{n} + S_B + F\right) - \frac{c_V Q(h_V + I_{Vp})}{2}\left[n\left(1 - \frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right] \\ &\quad - c_B I_{Vp} Dm - c_B h_B \frac{Q}{2} + \frac{pI_{Be}(Dm)^2}{2Q} - \frac{c_B I_{Bp}(Q - Dm)^2}{2Q}.\end{aligned} \tag{3.5}$$

To reduce the notations used by (3.5), we set $\Upsilon \equiv c_V(h_V + I_{Vp})$. We also replace $Q = DL$ and $D = D(p) = \gamma p^{-\beta}$ into the joint total annual profit function $\mathrm{JTP}(n, p, Q)$. Given that Z is a random variable with a known probability density function $f(z)$. Then we set the expected value of Z, $\mu = E(Z)$ and the expected value of (3.4), $\mathrm{EJTP}(n, p, L)$, is given as

$$\mathrm{EJTP}(n,p,L) = \begin{cases} \mathrm{EJTP}_1(n,p,L) = \mathrm{ETP}_V(n,p,L) + \mathrm{TP}_{B1}(p,L) & \text{if } L < m, \\ \mathrm{EJTP}_2(n,p,L) = \mathrm{ETP}_V(n,p,L) + \mathrm{TP}_{B2}(p,L) & \text{if } L \geq m, \end{cases} \tag{3.6}$$

where

$$\begin{aligned}&\mathrm{EJTP}_1(n,p,L) \\ &= \gamma p^{-\beta}\left\{p - c_V - \omega - c_R\mu + (pI_{Be} - c_B I_{Vp})m - \frac{L}{2}\left\{c_B h_B + pI_{Be} + \Upsilon\left[n\left(1 - \frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right]\right\}\right\} - \frac{1}{L}\left(\frac{S_V}{n} + S_B + F\right),\end{aligned} \tag{3.7}$$

$\mathrm{EJTP}_2(n,p,L)$

$$=\gamma p^{-\beta}\left\{p-c_V-\omega-c_R\mu+c_B\left(I_{Bp}-I_{Vp}\right)m-\frac{L}{2}\left\{c_B\left(h_B+I_{Bp}\right)+\Upsilon\left[n\left(1-\frac{1}{\lambda}\right)-1+\frac{2}{\lambda}\right]\right\}+\frac{(pI_{Be}-c_BI_{Bp})m^2}{2L}\right\}$$
$$-\frac{1}{L}\left(\frac{S_V}{n}+S_B+F\right). \tag{3.8}$$

4. Methodology

The objective of this paper is to find an optimal inventory policy to maximize the joint total annual profit between a vendor and a buyer.

4.1. Determination of the optimal number of shipments n for any given p and L

Firstly, taking the first-order and second-order partial derivatives of $\mathrm{EJTP}_i(n,p,L)$, for $i=1,2$, with respect to n, we obtain

$$\frac{\partial \mathrm{EJTP}(n,p,L)}{\partial n}=\begin{cases}\dfrac{\partial \mathrm{EJTP}_1(n,p,L)}{\partial n}\\[2mm] \dfrac{\partial \mathrm{EJTP}_2(n,p,L)}{\partial n}\end{cases}=\frac{-\gamma p^{-\beta}L\Upsilon}{2}\left(1-\frac{1}{\lambda}\right)+\frac{S_V}{n^2L},$$
$$\frac{\partial^2 \mathrm{EJTP}(n,p,L)}{\partial n^2}=\begin{cases}\dfrac{\partial^2 \mathrm{EJTP}_1(n,p,L)}{\partial n^2}\\[2mm] \dfrac{\partial^2 \mathrm{EJTP}_2(n,p,L)}{\partial n^2}\end{cases}=-\frac{2S_V}{n^3L}<0. \tag{4.1}$$

Therefore, for fixed p and L, $\mathrm{EJTP}_i(n,p,L)$ is strongly concave on $n>0$ for $i=1,2$. Thus, in each production run, determining the optimal number of shipments n^*, is simplified to obtain a global optimum.

4.2. Determination of the optimal replenishment time interval L for any given n and p

By taking the first-order and second-order partial derivatives of $\mathrm{EJTP}_i(n,p,L)$, for $i=1,2$, with respect to L, we have

$$\frac{\partial \mathrm{EJTP}_1(n,p,L)}{\partial L}=\frac{-\gamma p^{-\beta}}{2}\left\{c_Bh_B+pI_{Be}+\Upsilon\left[n\left(1-\frac{1}{\lambda}\right)-1+\frac{2}{\lambda}\right]\right\}+\frac{1}{L^2}\left(\frac{S_V}{n}+S_B+F\right), \tag{4.2}$$

$$\frac{\partial^2 \mathrm{EJTP}_1(n,p,L)}{\partial L^2}=-\frac{2}{L^3}\left(\frac{S_V}{n}+S_B+F\right)<0, \tag{4.3}$$

$$\frac{\partial \mathrm{EJTP}_2(n,p,L)}{\partial L}$$
$$=\frac{-\gamma p^{-\beta}}{2}\left\{c_B\left(h_B+I_{Bp}\right)+\Upsilon\left[n\left(1-\frac{1}{\lambda}\right)-1+\frac{2}{\lambda}\right]+\frac{(pI_{Be}-c_BI_{Bp})m^2}{L^2}\right\}+\frac{1}{L^2}\left(\frac{S_V}{n}+S_B+F\right), \tag{4.4}$$

$$\frac{\partial^2 \mathrm{EJTP}_2(n,p,L)}{\partial L^2}=-\frac{1}{L^3}\left[\gamma p^{-\beta}m^2\left(c_BI_{Bp}-pI_{Be}\right)+2\left(\frac{S_V}{n}+S_B+F\right)\right]. \tag{4.5}$$

Consequently, $\mathrm{EJTP}_1(n,p,L)$ is strongly concave on L for fixed n and p. So there exists a unique replenishment time interval value of L_1, which maximizes $\mathrm{EJTP}_1(n,p,L)$. The value of L_1 can be found by equating (4.2) to be zero, and we obtain

$$L_1 = \sqrt{\frac{(S_V/n + S_B + F)}{(\gamma p^{-\beta}/2)\{c_B h_B + pI_{Be} + \Upsilon[n(1-1/\lambda) - 1 + 2/\lambda]\}}}. \tag{4.6}$$

To make certain $L_1 < m$, we exchange (4.6) into inequality $L_1 < m$, and get that

$$\text{iff } \left(\frac{S_V}{n} + S_B + F\right) < \frac{\gamma p^{-\beta} m^2}{2}\left\{c_B h_B + pI_{Be} + \Upsilon\left[n\left(1 - \frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right]\right\}, \quad \text{then } L_1 < m. \tag{4.7}$$

Then substituting (4.6) into (3.7) and rearranging the result leads to

$$\begin{aligned}&\mathrm{EJTP}_1(n,p)\\ &\equiv \mathrm{EJTP}_1(n,p,L_1)\\ &= \gamma p^{-\beta}\left[p - c_V - \omega - c_R\mu + (pI_{Be} - c_B I_{Vp})m\right] - \sqrt{2\gamma p^{-\beta}\left(\frac{S_V}{n} + S_B + F\right)\left\{c_B h_B + pI_{Be} + \Upsilon\left[n\left(1-\frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right]\right\}}.\end{aligned} \tag{4.8}$$

From the inequality (4.7), we know that if $L_2 \geq m$, it means that

$$\left(\frac{S_V}{n} + S_B + F\right) \geq \frac{\gamma p^{-\beta} m^2}{2}\left\{c_B h_B + pI_{Be} + \Upsilon\left[n\left(1 - \frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right]\right\}. \tag{4.9}$$

Next, we need to check the second-order partial derivative of $\mathrm{EJTP}_2(n,p,L)$ for concavity with respect to L. The part $\Upsilon[n(1-1/\lambda) - 1 + 2/\lambda] = \Upsilon[((n-1)(\lambda-1)+1)/\lambda]$, where $n \geq 1$ and $\lambda > 1$, so it is certainly positive. Accordingly, it follows that

$$\gamma p^{-\beta} m^2 (c_B I_{Bp} - pI_{Be}) + 2\left(\frac{S_V}{n} + S_B + F\right) \geq \gamma p^{-\beta} m^2 \left\{c_B(h_B + I_{Bp}) + \Upsilon\left[n\left(1 - \frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right]\right\} > 0, \tag{4.10}$$

and the polynomial (4.5) is negative. Therefore, $\mathrm{EJTP}_2(n,p,L)$ is also strongly concave on L for fixed n and p. Similarly, we can obtain an optimum of L_2 which maximizes $\mathrm{EJTP}_2(n,p,L)$. Solving for L_2 by equating (4.4) to be zero, we have

$$L_2 = \sqrt{\frac{(S_V/n + S_B + F) + (\gamma p^{-\beta} m^2/2)(c_B I_{Bp} - pI_{Be})}{(\gamma p^{-\beta}/2)\{c_B(h_B + I_{Bp}) + \Upsilon[n(1-1/\lambda) - 1 + 2/\lambda]\}}}. \tag{4.11}$$

To make certain $L_2 \geq m$, we exchange (4.11) into inequality $L_2 \geq m$, and get that

$$\text{iff } \left(\frac{S_V}{n} + S_B + F\right) \geq \frac{\gamma p^{-\beta} m^2}{2}\left\{c_B h_B + pI_{Be} + \Upsilon\left[n\left(1 - \frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right]\right\}, \quad \text{then } L_2 \geq m. \tag{4.12}$$

Substituting (4.11) into (3.8) and rearranging

$$\begin{aligned}\mathrm{EJTP}_2(n,p)&\\ &\equiv \mathrm{EJTP}_2(n,p,L_2)\\ &=\gamma p^{-\beta}\left[p-c_V-\omega-c_R+c_B(I_{Bp}-I_{Vp})m\right]\\ &\quad-\sqrt{2\gamma p^{-\beta}\left[\left(\frac{S_V}{n}+S_B+F\right)+\frac{\gamma p^{-\beta}m^2}{2}(c_BI_{Bp}-pI_{Be})\right]\left\{c_B(h_B+I_{Bp})+\Upsilon\left[n\left(1-\frac{1}{\lambda}\right)-1+\frac{2}{\lambda}\right]\right\}}.\end{aligned}\tag{4.13}$$

Hence, the above processes on L lead to the following theorem.

Theorem 4.1. *For any given n and p, we can get the following results.*

(i) *If* $(S_V/n+S_B+F)<(\gamma p^{-\beta}m^2/2)\{c_Bh_B+pI_{Be}+\Upsilon[n(1-1/\lambda)-1+2/\lambda]\}$, *then* $L^*=L_1$.

(ii) *If* $(S_V/n+S_B+F)\geq(\gamma p^{-\beta}m^2/2)\{c_Bh_B+pI_{Be}+\Upsilon[n(1-1/\lambda)-1+2/\lambda]\}$, *then* $L^*=L_2$.

(iii) *If* $(S_V/n+S_B+F)=(\gamma p^{-\beta}m^2/2)\{c_Bh_B+pI_{Be}+\Upsilon[n(1-1/\lambda)-1+2/\lambda]\}$, *then* $L^*=m$.

Proof. The above processes on L imply that Theorem 4.1 holds. □

4.3. Determination of the optimal selling price p

According to Theorem 4.1, we set a function of p, $\psi(p)$, as a distinction function which is given to be

$$\psi(p)=\frac{\gamma p^{-\beta}m^2}{2}\left\{c_Bh_B+pI_{Be}+\Upsilon\left[n\left(1-\frac{1}{\lambda}\right)-1+\frac{2}{\lambda}\right]\right\}.\tag{4.14}$$

$\psi(p)$ is a monotonically decreasing function of p, and p is a monotonic variable, where given any $p^+>p^-$ such that $\psi(p^+)<\psi(p^-)$, because

$$\begin{aligned}\frac{d\psi(p)}{dp}&=\frac{-\gamma p^{-\beta-1}m^2}{2}\left\{\beta\left\{c_Bh_B+\Upsilon\left[n\left(1-\frac{1}{\lambda}\right)-1+\frac{2}{\lambda}\right]\right\}+(\beta-1)pI_{Be}\right\}\\ &=\frac{-\gamma p^{-\beta-1}m^2}{2}\left\{\beta\left\{c_Bh_B+\Upsilon\left[\frac{(n-1)(\lambda-1)+1}{\lambda}\right]\right\}+(\beta-1)pI_{Be}\right\}<0,\end{aligned}\tag{4.15}$$

where $n\geq1$, $\lambda>1$ and $\beta>1$.

Utilizing the results in Theorem 4.1, we set p_0 such that

$$\left(\frac{S_V}{n}+S_B+F\right)=\psi(p_0)=\frac{\gamma p_0^{-\beta}m^2}{2}\left\{c_Bh_B+p_0I_{Be}+\Upsilon\left[n\left(1-\frac{1}{\lambda}\right)-1+\frac{2}{\lambda}\right]\right\}.\tag{4.16}$$

Then for any given p which is substituting into $\psi(p)$, we get

$$\left(\frac{S_V}{n}+S_B+F\right)\begin{cases}<\psi(p), & \text{if } p<p_0\\ \geq\psi(p), & \text{if } p\geq p_0.\end{cases}\tag{4.17}$$

By comparing (4.7), (4.12), and (4.17), the following results can be yielded

$$\begin{aligned} &\text{iff } p < p_0, \quad \text{then } L_1 < m. \\ &\text{iff } p \geq p_0, \quad \text{then } L_2 \geq m. \end{aligned} \tag{4.18}$$

Consequently, we know from (3.6), (4.8), (4.13), and (4.18) that

$$\text{EJTP}(n,p) = \begin{cases} \text{EJTP}_1(n,p) = \text{EJTP}_1(n,p,L_1) & \text{if } p < p_0, \\ \text{EJTP}_2(n,p) = \text{EJTP}_2(n,p,L_2) & \text{if } p \geq p_0, \end{cases} \tag{4.19}$$

where n is fixed.

Solving for the optimal selling price $p*$, by taking the first-order partial derivative of (4.8) with respect to p and equating the result to be zero, we obtain

$$\begin{aligned} \frac{\partial \text{EJTP}_1(n,p)}{\partial p} =& \gamma p^{-\beta}(1-\beta)(1+I_{Be}m) + \beta\gamma p^{-\beta-1}(c_V + \omega + c_R\mu + c_B I_{Vp} m) \\ &+ \sqrt{\frac{\gamma p^{-\beta}(S_V/n + S_B + F)}{2}} \times \frac{(\beta/p)\{c_B h_B + pI_{Be} + \Upsilon[n(1-1/\lambda) - 1 + 2/\lambda]\} - I_{Be}}{\sqrt{c_B h_B + pI_{Be} + \Upsilon[n(1-1/\lambda) - 1 + 2/\lambda]}} \\ =& 0. \end{aligned} \tag{4.20}$$

Then we need to verify the second-order partial derivative condition for concavity, as

$$\begin{aligned} &\frac{\partial^2 \text{EJTP}_1(n,p)}{\partial p^2} \\ &= -\beta\gamma p^{-\beta-2}[p(1-beta)(1+I_{Be}m) + (\beta+1)(c_V + \omega + c_R\mu + c_B I_{Vp} m)] - \sqrt{\frac{\gamma p^{-\beta-4}(S_V/n + S_B + F)}{8}} \\ &\times \left\{ (\beta+1)pI_{Be}\left\{2\beta\left\{c_B h_B + pI_{Be} + \Upsilon\left[n\left(1-\frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right]\right\} - (\beta+1)pI_{Be}\right\} \right. \\ &\left. + \beta(\beta+2)\left\{c_B h_B + \Upsilon\left[n\left(1-\frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right]\right\}^2 \right\} \left\{c_B h_B + pI_{Be} + \Upsilon\left[n\left(1-\frac{1}{\lambda}\right) - 1 + \frac{2}{\lambda}\right]\right\}^{-3/2} < 0. \end{aligned} \tag{4.21}$$

Likewise, taking the first-order partial derivative of (4.13) with respect to p and equating the result to be zero, we get

$$\begin{aligned} \frac{\partial \text{EJTP}_2(n,p)}{\partial p} =& \beta\gamma p^{-\beta-1}[c_V + \omega + c_R\mu + c_B(I_{Vp} - I_{Bp})m] + \gamma p^{-\beta}(1-\beta) \\ &+ \left\{ \frac{\beta}{p}\left(\frac{S_V}{n} + S_B + F\right) + \gamma p^{-\beta-1} m^2 \left[\beta c_B I_{Bp} + \left(\frac{1}{2} - \beta\right) pI_{Be}\right] \right\} \\ &\times \sqrt{\frac{(\gamma p^{-\beta}/2)\{c_B(h_B + I_{Bp}) + \Upsilon[n(1-1/\lambda) - 1 + 2/\lambda]\}}{(S_V/n + S_B + F) + (\gamma p^{-\beta} m^2/2)(c_B I_{Bp} - pI_{Be})}} \\ =& 0. \end{aligned} \tag{4.22}$$

Step 1: set $n = 1$.
Step 2: determine the p_0 by solving (4.16).
Step 3: if there exists a p_1 where $p_1 \leq p_0$, and satisfies both the first-order condition as in (4.20) and the second-order condition as in (4.21), then we compute $L_1(p_1)$ by (4.6) and $\text{EJTP}_1(n, p_1, L_1(p_1))$ by (4.8). If not, we set $\text{EJTP}_1(n, p_1, L_1(p_1)) = 0$.
Step 4: if there exists a p_2 where $p_2 \geq p_0$, and satisfies both the first-order condition as in (4.22) and the second-order condition as in (4.23), then we compute $L_2(p_2)$ by (4.11) and $\text{EJTP}_2(n, p_2, L_2(p_2))$ by (4.13). If not, we set $\text{EJTP}_2(n, p_2, L_2(p_2)) = 0$.
Step 5: if $\text{EJTP}_1(n, p_1, L_1(p_1)) \geq \text{EJTP}_2(n, p_2, L_2(p_2))$. Set $\text{EJTP}(n, p[n], L[n]) = \text{EJTP}_1(n, p_1, L_1(p_1))$, then $(p[n], L[n])$ is an optimal solution for a given n. If not, $\text{EJTP}(n, p[n], L[n]) = \text{EJTP}_2(n, p_2, L_2(p_2))$.
Step 6: set $n = n + 1$, repeat steps 2–5 to obtain $\text{EJTP}(n, p[n], L[n])$.
Step 7: if $\text{EJTP}(n, p[n], L[n]) \geq \text{EJTP}(n-1, p[n-1], L[n-1])$, go to step 6. If not, go to step 8 and stop.
Step 8: set $\text{EJTP}(n^*, p^*, L^*) = \text{EJTP}(n-1, p[n-1], L[n-1])$, so (n^*, p^*, L^*) is an optimal solution. Consequently, the buyer's optimal order quantity per order is $Q^* = D(p^*)L^*$.

Algorithm 1

The second-order condition for concavity that we need to verify is

$$\begin{aligned}
&\frac{\partial^2 \text{EJTP}_2(n,p)}{\partial p^2} \\
&= -\gamma p^{-\beta-2}\Bigg\{\beta\{(\beta+1)\left[c_V + \omega + c_R\mu + c_B\left(I_{Vp} - I_{Bp}\right)m\right] + (1-\beta)p\} \\
&\qquad + \frac{\sqrt{\gamma p^{-\beta}\{c_B\left(h_B + I_{Bp}\right) + \Upsilon\left[n(1-1/\lambda) - 1 + 2/\lambda\right]\}}}{\left[2\left(S_V/n + S_B + F\right) + m^2\left(c_B I_{Bp} - pI_{Be}\right)\right]^{3/2}} \\
&\qquad \times \Bigg\{\frac{\beta(\beta+2)}{\gamma p^{-\beta}}\left(\frac{S_V}{n} + S_B + F\right)^2 + 3\beta m^2\left(\frac{S_V}{n} + S_B + F\right)\left[c_B(\beta+1)I_{Bp} - \beta p I_{Be}\right] \\
&\qquad + \gamma p^{-\beta} m^4\left[\left(\beta^2 - \frac{1}{4}\right)\left(pI_{Be}\right)^2 + \beta(2\beta+1)c_B I_{Bp} p I_{Be} - \beta(\beta+1)\left(c_B I_{Bp}\right)^2\right]\Bigg\}\Bigg\} \\
&< 0.
\end{aligned} \tag{4.23}$$

4.4. Optimal solution procedure

Thus, we can use the following solution procedure to find optimal values n, p, and L for this model. The solution procedure is commonly known as dichotomy, as in Algorithm 1.

5. Numerical examples

Example 5.1. Consider an inventory situation with the following parametric values partially adopted in Ouyang et al. [13] and Salameh and Jaber [22]:

(i) scaling factor $\gamma = 100000$,

(ii) index of price elasticity $\beta = 1.5$,

(iii) ratio between the production rate and the demand rate $\lambda = 1.5$,

(iv) purchasing cost for the buyer $c_B = \$4.5/\text{unit}$,

(v) ordering cost for the buyer $S_B = \$10/\text{order}$,

(vi) buyer's unit holding cost rate $h_B = 0.111$,

(vii) buyer's capital opportunity cost $I_{Bp} = 0.08/\$/\text{yr}$,

(viii) buyer's interest earned rate $I_{Be} = 0.06/\$/\text{yr}$,

(ix) transportation cost $F = \$50/\text{shipment}$,

(x) production cost for the vendor $c_V = \$2.2/\text{unit}$,

(xi) setup cost per production run for the vendor $S_V = \$350/\text{setup}$,

(xii) vendor's unit holding cost rate $h_V = 0.046$,

(xiii) vendor's capital opportunity cost $I_{Vp} = 0.03/\$/\text{yr}$,

(xiv) inspecting cost $\omega = \$0.5/\text{unit}$,

(xv) repair cost per imperfect quality item $c_R = \$2/\text{unit}$.

The percentage defective random variable, Z, is uniformly distributed with its probability density function (PDF) as

$$f(z) = \begin{cases} 25, & 0 \le z \le 0.04 \\ 0, & \text{otherwise.} \end{cases} \tag{5.1}$$

Therefore, $\mu = E[Z] = \int_0^{0.04} 25z\,dz = 0.02$.

The above solution algorithm is applied to get the computational results for various values of permissible delay period m as shown in Table 1.

Table 1 shows that (1) the expected joint total annual profit increases when the permissible delay period m increases, (2) the optimal selling price p^* and the optimal replenishment time interval L^* are decreasing with the increasing of permissible delay period m, (3) as the annual demand $D(p^*)$ is increasing with the decreasing of p^*, the buyer's expected total annual profit increases as well as the expected joint total annual profit, and (4) the optimal order quantity Q^* decreases with the increasing of permissible delay period within the range of $0 < m \le 70$. Generally speaking, a longer permissible delay period offered may motivate the buyer to carry out frequent shipments in small batches. It also can shorten the replenishment time interval to utilize the credit period in the profit increasing and cost reduction more and more. In addition, it also can be observed from Table 1 that when $m < 44$, the vendor's expected total annual profit follows the value of permissible delay period m increasing. But when $m \ge 44$, the expected total annual profit of the vendor decreases as the value of permissible delay

Table 1: Optimal solutions for various values of permissible delay period $m(\lambda = 1.50)$.

m (day)	n^*	p_0	p^*	L^* (day)	$D(p^*)$	Q^*	n^*Q^*	Profit ($/yr) Vendor	Buyer	Joint
0	10	—	$p_2 = 8.6191$	$L_2 = 65.9521$	3951.9107	714.0734	7140.7344	6542.7743	15639.3831	22182.1574
5	10	0.2313	$p_2 = 8.6097$	$L_2 = 65.8780$	3958.3844	714.4393	7144.3930	6546.5177	15648.1525	22194.6702
10	10	0.5903	$p_2 = 8.6007$	$L_2 = 65.7660$	3964.5993	714.3443	7143.4434	6549.8114	15658.0354	22207.8468
15	10	1.0290	$p_2 = 8.5920$	$L_2 = 65.6157$	3970.6225	713.7949	7137.9488	6552.7706	15668.9163	22221.6869
20	10	1.5361	$p_2 = 8.5837$	$L_2 = 65.4278$	3976.3830	712.7838	7127.8381	6555.2744	15680.9170	22236.1914
25	10	2.1074	$p_2 = 8.5758$	$L_2 = 65.2024$	3981.8788	711.3094	7113.0936	6557.3194	15694.0429	22251.3623
30	10	2.7421	$p_2 = 8.5683$	$L_2 = 64.9392$	3987.1080	709.3688	7093.6881	6558.9020	15708.3011	22267.2031
40	10	4.2066	$p_2 = 8.5545$	$L_2 = 64.2988$	3996.7599	704.0736	7040.7364	6560.6634	15740.2516	22300.9150
41	10	4.3680	$p_2 = 8.5532$	$L_2 = 64.2263$	3997.6711	703.4398	7034.3977	6560.7450	15743.6916	22304.4366
42	10	4.5321	$p_2 = 8.5519$	$L_2 = 64.1521$	3998.5827	702.7877	7027.8768	6560.8266	15747.1593	22307.9859
43	10	4.6991	$p_2 = 8.5506$	$L_2 = 64.0763$	3999.4946	702.1173	7021.1732	6560.9080	15750.6547	22311.5627
44	10	4.8689	$p_2 = 8.5494$	$L_2 = 63.9994$	4000.3367	701.4221	7014.2213	6560.8706$^{(b)}$	15754.2967	22315.1673
45	10	5.0417	$p_2 = 8.5482$	$L_2 = 63.9208$	4001.1791	700.7085	7007.0854	6560.8330	15757.9668	22318.7998
50	10	5.9502	$p_2 = 8.5424$	$L_2 = 63.5041$	4005.2548	696.8497	6968.4967	6560.4028	15776.9794	22337.3822
60	11	8.4755	$p_2 = 8.5341$	$L_2 = 60.2724$	4011.0993	662.3522	7285.8737	6556.3448	15820.4483	22376.7931
70	11	11.0359	$p_1 = 8.5309$	$L_1 = 60.2343$	4013.3564	662.3060$^{(a)}$	7285.3655	6545.3169	15872.9112	22418.2281
80	11	14.0380	$p_1 = 8.5280$	$L_1 = 60.2221$	4015.4037	662.5092	7287.6008	6533.9086	15925.7575	22459.6661
90	11	17.5552	$p_1 = 8.5250$	$L_1 = 60.2094$	4017.5234	662.7195	7289.9147	6522.6064	15978.4997	22501.1061

(a) When $0 < m \leq 70$, the optimal order quantity Q^* is negatively correlated to the length of the permissible delay period m.
(b) When $m < 44$, the vendor's expected total annual profit is positively correlated to the length of the permissible delay period m but as $m \geq 44$, it is reverse.

period m increases. These results indicate that the buyer can always profit from the permissible delay period. For the vendor, he can also profit from the permissible delay in payments strategy while the credit period of time is not longer than 44 days. But on the contrary, if the permissible delay period m is greater than 44 days, the vendor's expected total annual profit decreases through his sales revenue by permitting the buyer a credit period of time cannot disburse his opportunity cost. Based on the above discussions, it illustrates that applying the permissible delay in payments strategy in an integrated inventory model would advance the profit increasing and cost reduction.

Proceeding to the next, we compare the proposed model herein with the model established by Ouyang et al. [13]. The two models are mainly different from considering imperfect quality items or not. These comparison results are presented in Table 2.

Clearly, it is seen that imperfect quality items cause a significant profit loss. The improvement of the joint profit is greater than 10%. Besides, we also compare the relevant profit of the vendor and the buyer in the proposed model with Ouyang et al. [13] further. The results show that the vendor's profit is downward obviously due to the effect of imperfect quality items. His profit improvement is very big and the improved range is greater than 45%. Table 1 reveals that the optimal selling prices p^*s in the situations where the values of permissible delay period $m \in \{0, 10, 30, 60\}$ are all higher than them as in Ouyang et al. [13]. Hence, this causes the annual demand $D(p^*)$ and the optimal order quantity Q^* to be smaller than them as in Ouyang et al. [13], this result also induces the damage of the vendor's sales revenue. Furthermore, the process unreliability consideration between the vendor and the buyer will

Table 2: Comparison results with Ouyang et al. (2005).

m (day)	Profit ($/yr)								
	Joint			Vendor			Buyer		
	This paper	Ouyang et al.	Improved (%)	This paper	Ouyang et al.	Improved (%)	This paper	Ouyang et al.	Improved (%)
0	22182.1574	24691.0000	−10.16	6542.7743	12100.0000	−45.93	15639.3831	12591.0000	24.21
10	22207.8468	24725.0000	−10.18	6549.8114	12127.0000	−45.99	15658.0354	12598.0000	24.29
30	22267.2031	24799.0000	−10.21	6558.9020	12157.0000	−46.05	15708.3011	12642.0000	24.25
60	22376.7931	24921.0000	−10.21	6556.3448	12138.0000	−45.98	15820.4483	12783.0000	23.76

$\lambda = 1.50$.

Table 3: Optimal solutions for various values of λ ($m = 30$).

λ	n^*	p_0	p^*	L^* (day)	$D(p^*)$	Q^*	n^*Q^*	Profit ($/yr) Vendor	Buyer	Joint
1.01	61	2.8477	$p_2 = 8.4577$	$L_2 = 60.9404$	4065.5713	678.7876	41406.0421	6986.0330	15571.6602	22557.6932
1.10	20	2.8641	$p_2 = 8.5068$	$L_2 = 61.9366$	4030.4234	683.9196	13678.3915	6794.9584	15632.9216	22427.8800
1.50	10	2.7421	$p_2 = 8.5683$	$L_2 = 64.9392$	3987.1080	709.3688	7093.6881	6558.9020	15708.3011	22267.2031
2.00	8	2.6533	$p_2 = 8.5947$	$L_2 = 66.8279$	3968.7516	726.6397	5813.1179	6459.0245	15739.6967	22198.7212
3.00	7	2.6202	$p_2 = 8.6158$	$L_2 = 67.7510$	3954.1814	733.9719	5137.8034	6380.2218	15764.2464	22144.4682

incur the vendor to bear the warranty cost. So the vendor's profit in the proposed model is smaller than it as in Ouyang et al. [13]. But, the buyer's profit is upward, and his profit improvement is greater than 23%. The results reveal that the buyer's profit increment is from his profit raise owing to the higher optimal selling price p^* which can pay for the total of his demand decrement owing to the higher optimal selling price p^* and the inspecting cost incurred in finding and expelling imperfect quality items. Therefore, the buyer's profit in the proposed model is larger than it as in Ouyang et al. [13]. Then from the above discussions in Table 2, it demonstrates that the proposed model produces a significant profit loss when comparing with the joint total annual profit without considering imperfect quality items. These results have really met the truth.

Example 5.2. We take the same values for the parameters as in Example 5.1. Suppose the value of permissible delay period $m = 30$, we investigate the effect of the ratio between the production rate and the demand rate, λ. Similarly, we also compare the proposed model herein with the model of Ouyang et al. [13]. Following the above solution procedure, the computational results for various values of the ratio λ are presented in Table 3.

Table 3 reveals that the expected total annual profit of the vendor and the whole integrated inventory model increase as the value of the ratio λ is close to 1. On the contrary, the buyer's profit decreases. These results are the same as the conclusions in Ouyang et al. [13]. The results imply that if the JIT cooperation between the vendor and the buyer could be implemented successfully, the vendor's profit and the joint profit will increase following that the vendor can get the real time demand rate through the buyer and adjust his production rate to the demand rate. Comparison results between the proposed model and the model of Ouyang et al. [13] are shown in Table 4. The results reveal that the profit improvement of the joint profit is greater than 10%. Obviously, imperfect quality items can lead to a noticeable profit loss.

Table 4: Comparison results with Ouyang et al. (2005).

λ	Profit ($/yr)								
	Joint			Vendor			Buyer		
	This paper	Ouyang et al.	Improved (%)	This paper	Ouyang et al.	Improved (%)	This paper	Ouyang et al.	Improved (%)
1.01	22557.6932	25140.0000	−10.27	6986.0330	12758.0000	−45.24	15571.6602	12382.0000	25.76
1.10	22427.8800	24988.0000	−10.25	6794.9584	12489.0000	−45.59	15632.9216	12499.0000	25.07
1.50	22267.2031	24799.0000	−10.21	6558.9020	12157.0000	−46.05	15708.3011	12642.0000	24.25
2.00	22198.7212	24719.0000	−10.20	6459.0245	12017.0000	−46.25	15739.6967	12702.0000	23.92
3.00	22144.4682	24655.0000	−10.18	6380.2218	11906.0000	−46.41	15764.2464	12749.0000	23.65

$m = 30$ days.

6. Conclusions

This paper investigates a production/inventory situation which producing process would go out of control under permissible delay in payments. In this research, we assume that in the vendor's production process, the imperfect quality items are reworked immediately as they are found and meantime the vendor must bear the repair cost. This new proposed model herein shows a different thought on inventory modeling. The expected joint total annual profit function has been derived. Then by analyzing this derived function, we can obtain the unique closed-form optimal solution for the replenishment time interval and develop a simple solution procedure to determine the buyer's optimal selling price, order quantity, and the number of shipments per production runs from the vendor to the buyer. Finally, the numerical examples adopted in the Ouyang et al. [13] and Salameh and Jaber [22] explain the solution algorithm. These results reveal that applying the permissible delay in payments strategy between the vendor and the buyer can promote the profit increasing and cost reduction. They also indicate that the successful implementation of JIT cooperation in an integrated inventory model leads to the profit rise of the whole inventory model. Besides, the proposed model generates an impressive profit loss when compared with the joint total annual profit without incorporating imperfect quality items into consideration.

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