Research Article

Some Exact Solutions of Boundary Layer Flows along a Vertical Plate with Buoyancy Forces Combined with Lorentz Forces under Uniform Suction

Asterios Pantokratoras

Department of Civil Engineering, School of Engineering, Democritus University of Thrace, 67100 Xanthi, Greece

Correspondence should be addressed to Asterios Pantokratoras, apantokr@civil.duth.gr

Exact analytical solutions of boundary layer flows along a vertical porous plate with uniform suction are derived and presented in this paper. The solutions concern the Blasius, Sakiadis, and Blasius-Sakiadis flows with buoyancy forces combined with either MHD Lorentz or EMHD Lorentz forces. In addition, some exact solutions are presented specifically for water in the temperature range of $0°C \leq T \leq 8°C$, where water density is nearly parabolic. Except for their use as benchmarking means for testing the numerical solution of the Navier-Stokes equations, the presented exact solutions with EMHD forces have use in flow separation control in aeronautics and hydronautics, whereas the MHD results have applications in process metallurgy and fusion technology. These analytical solutions are valid for flows with strong suction.

1. Introduction

Free convection along a vertical plate with mass transfer at the wall has been studied by many authors in the past. Eichhorn [1] was the first to study the effect of suction and injection on free convective flow. He considered power-law variation of plate temperature and transpiration velocity under which self-similar solutions of the governing equations are possible. For the case of isothermal plate with uniform blowing or suction, similarity does not exist. For the latter problem, Sparrow and Cess [2] provided approximate series solutions valid near the plate leading edge for Pr = 0.72. This problem was considered in more detail by Merkin [3] who obtained asymptotic solutions, valid at large distances from the leading edge. The next-order corrections to the boundary layer solution for this problem, concerning gases, were obtained by Clarke [4] who did not invoke the usual Boussinesq approximation. The solutions for strong suction and blowing on general body shapes which admit a similarity solution have been given by Merkin [5]. Parikh et al. [6]

studied both numerically and experimentally the problem of air (Pr = 0.7) free convection over an isothermal porous vertical plate with uniform transpiration taking into account the air variable physical properties. Minkowycz and Sparrow [7] using the local nonsimilarity method presented solutions for a wide range of Pr numbers. Vedhanayagam et al. [8] presented a transformation of the equations for general blowing and wall temperature variations, as well as results for the isothermal plate with uniform blowing. A solution to the constant plate temperature with uniform air blowing, based on the film model, has been derived by Brouwers [9]. In a subsequent paper, Brouwers [10], using the film model, derived a thermal correction factor and a novel friction correction factor which were applied to free convection along a vertical porous plate. The problem of blowing and suction on the free convection over a vertical plate with a given wall heat flux has been considered by Chaudhary and Merkin [11] who presented results valid for Pr = 1. Merkin [12] considered again the problem of free convection flow over a vertical plate with prescribed temperature, and presented results for variable transpiration velocities for Pr 1 and 7.

In all of the above works, the buoyant force is produced by the temperature difference between the plate and the ambient fluid. Another kind of vertical force which resembles the buoyant force is the electromagnetohydrodynamic (EMHD) Lorentz force which acts parallel to the plate either assisting or opposing the flow. The EMHD Lorentz force can be generated by a stripwise arrangement of flush mounted electrodes and permanent magnets. Gailitis and Lielausis [13] were probably the first to propose the use of the EMHD Lorentz force for flow control over a plate. The idea of using the EMHD Lorentz force for flow control by Gailitis and Lielausis was later abandoned and only recently attracted new attention [14–17]. In addition, in last years much investigation on flow control using the EMHD Lorentz force was conducted at the Rossendorf Institute and the Institute for Aerospace Engineering in Dresden, Germany [18–22].

Magnetohydrodynamics (MHD) is the field of fluid mechanics that encompasses the phenomena arising when a magnetic field is applied to an electrically conducting fluid. Air, water, and especially liquid metals (lithium, mercury, and sodium) are electrically conducting fluids. The effect of an applied magnetic field on heat transfer in external flows has been investigated mainly for the cases of flat plate boundary layer and blunt body stagnation point flows. The works published in these areas appeared in the late 1950's and early 1960's with application to space-vehicle surface heating upon reentry. MHD is also applied to fusion technology [23].

The problem of flow along a vertical, stationary, isothermal plate of an electrically conducting fluid under a horizontal magnetic field is a classical problem in magnetohydrodynamics and has been treated for the first time by Sparrow and Cess [24]. Riley [25] and Kuiken [26] have reexamined the problem in order to give exact solutions, but their attempts to use the method of matched asymptotic expansions encountered difficulties. The first complete exact results for this problem have been given by Wilks [27] for Pr = 0.72.

In all of the above works, the vertical plate and the ambient fluid were motionless; that is, the problem treated was pure free convection. The problem of fluid flow along a stationary, impermeable, horizontal plate situated in a fluid stream moving with constant velocity is a classical problem of fluid mechanics that has been solved for the first time in 1908 by Blasius [28]. In this problem, the fluid motion is produced by the free stream. A similar problem occurs when the plate moves with constant velocity in a calm fluid. This problem has been treated for the first time by Sakiadis [29]. MHD flows along a moving plate in viscoelastic and micropolar fluids are treated in [30–34]. The combination of a moving vertical plate within
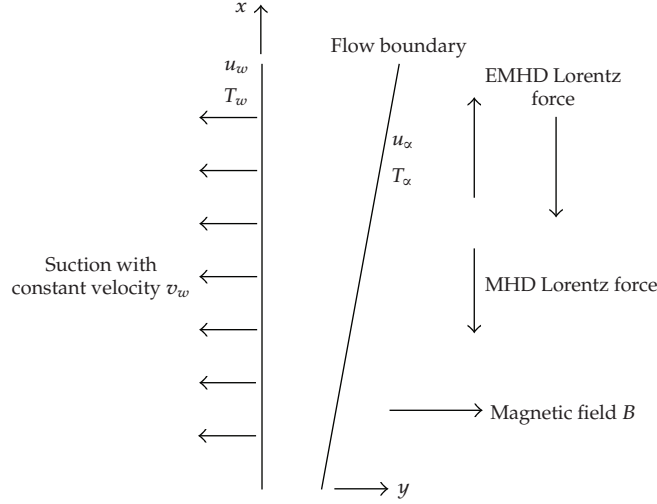
Figure 1: The flow configuration and coordinate system.

a vertical free stream with the existence of a vertical buoyancy force is called here Sakiadis-Blasius-buoyant flow.

The purpose of the present paper is to present some new exact analytical solutions for the Sakiadis-Blasius-buoyant flow over a vertical plate with uniform suction with either EMHD Lorentz or MHD Lorentz forces. Exact solutions of the Navier-Stokes equations are important for two reasons [35]. Owing to their uniform validity, the basic phenomena described by them can be more closely studied. In addition, the exact solutions serve as standards for checking the accuracies of the many approximate methods, whether they are numerical, asymptotic, or empirical. Current computer technology makes the complete numerical integration of the Navier-Stoles equations feasible. However, the accuracy of the results can only be ascertained by comparison with an exact solution.

2. The mathematical model

Consider the flow along a vertical plate with u and v denoting, respectively, the velocity components in the x and y directions, where x is the coordinate along the plate and y is the coordinate perpendicular to x (see Figure 1). For steady two-dimensional flow, the boundary layer equations with constant fluid properties and linear relationship between density and temperature are as follows:

continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty), \tag{2.2}$$

energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \lambda\frac{\partial^2 T}{\partial y^2}, \tag{2.3}$$

where ν is the fluid kinematic viscosity, g is the gravitational acceleration, β is the volumetric expansion coefficient, T is the fluid temperature, and λ is the fluid thermal diffusivity.

When the suction velocity is very strong, (2.1)–(2.3) take the following forms [36, page 297]:

continuity equation:

$$\frac{\partial v}{\partial y} = 0, \tag{2.4}$$

momentum equation:

$$v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty), \tag{2.5}$$

energy equation:

$$v\frac{\partial T}{\partial y} = \lambda\frac{\partial^2 T}{\partial y^2}. \tag{2.6}$$

It follows from (2.4) that $v = v_w$ = constant and the momentum and energy equations take the following forms:

momentum equation:

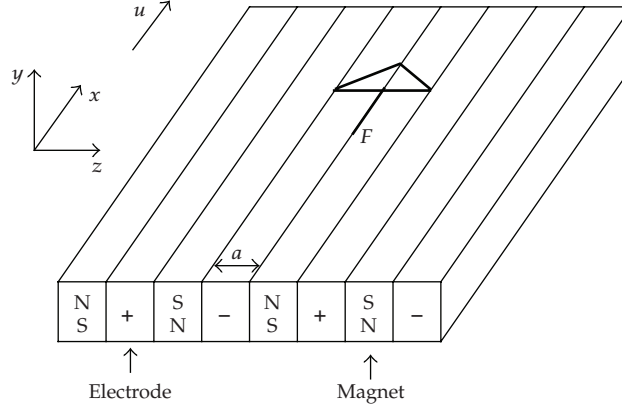$$\nu\frac{\partial^2 u}{\partial y^2} - v_w\frac{\partial u}{\partial y} = -g\beta(T - T_\infty), \tag{2.7}$$

energy equation:

$$\lambda\frac{\partial^2 T}{\partial y^2} - v_w\frac{\partial T}{\partial y} = 0. \tag{2.8}$$

For combined Sakiadis-Blasius flow, the boundary conditions are as follows:

$$\begin{aligned} \text{at } y = 0: \quad u = u_w, \qquad v = v_w, \qquad T = T_w, \\ \text{as } y \longrightarrow \infty, \qquad u = u_\infty, \qquad T = T_\infty. \end{aligned} \tag{2.9}$$

In the present work, the suction velocity v_w is always negative.

Figure 2: Arrangement of electrodes and magnets for the creation of an EMHD Lorentz force F in the flow along a flat plate [20].

The energy equation (2.8) is independent of the momentum equation, and its solution [36, page 207] is

$$\frac{T - T_\infty}{T_w - T_\infty} = \exp\left(\frac{v_w y}{\lambda}\right). \tag{2.10}$$

3. Results and discussion

3.1. Sakiadis-Blasius flow with buoyancy forces and EMHD Lorentz forces

It is assumed that, except for the buoyancy force, an EMHD Lorentz force acts parallel to the plate. The EMHD Lorentz force can be generated by a stripwise arrangement of flush mounted electrodes and permanent magnets as sketched in Figure 2. For more information see Weier [20]. Then, the momentum equation (2.7) takes the following form:

$$\nu\frac{\partial^2 u}{\partial y^2} - v_w\frac{\partial u}{\partial y} = -g\beta(T - T_\infty) - \frac{\pi j_0 M_0}{8\rho_\infty}\exp\left(-\frac{\pi}{a}y\right), \tag{3.1}$$

where j_0 is the applied current density, M_0 is the magnetization of the permanent magnets, α is the width of magnets and electrodes, and ρ_∞ is the ambient fluid density. It should be noted here that the EMHD Lorentz force can either assist or oppose the flow and is independent of the flow field. The last term in the momentum equation is the EMHD Lorentz force which assists the flow and decreases exponentially with y. Substituting the temperature difference from (2.10) into (3.1), we have

$$\nu\frac{\partial^2 u}{\partial y^2} - v_w\frac{\partial u}{\partial y} = -g\beta(T_w - T_\infty)\exp\left(\frac{v_w}{\lambda}y\right) - \frac{\pi j_0 M_0}{8\rho_\infty}\exp\left(-\frac{\pi}{a}y\right). \tag{3.2}$$

Equation (3.2) is a linear differential equation of second order and has the following exact analytical solution for the Sakiadis-Blasius-buoyant-EMHD flow:

$$\frac{u - u_\infty}{u_w - u_\infty} = (1 + G + Z)\exp\left(\frac{v_w}{v}y\right) - G\exp\left(\frac{v_w}{\lambda}y\right) - Z\exp\left(-\frac{\pi}{a}y\right), \qquad (3.3)$$

where G is the buoyancy parameter:

$$G = \frac{\lambda g\beta(T_w - T_\infty)}{(u_w - u_\infty)v_w^2(\mathrm{Pr}-1)} \qquad (3.4)$$

and Z is the Lorentz parameter:

$$Z = \frac{j_0 M_0 a^2}{8\rho_\infty(u_w - u_\infty)(\pi v + av_w)}. \qquad (3.5)$$

The Prandtl number is

$$\mathrm{Pr} = \frac{v}{\lambda}. \qquad (3.6)$$

Both parameters G and Z are dimensionless. The absolute wall shear stress is

$$\tau_w = \rho_\infty(u_w - u_\infty)v_w - \frac{\mu g\beta(T_w - T_\infty)}{v_w\,\mathrm{Pr}} + \frac{j_0 M_0 a}{8}, \qquad (3.7)$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho_\infty(u_w - u_\infty)v_w} = 1 + G(1 - \mathrm{Pr}) + Z\left(\frac{\pi}{\mathrm{Re}} + 1\right), \qquad (3.8)$$
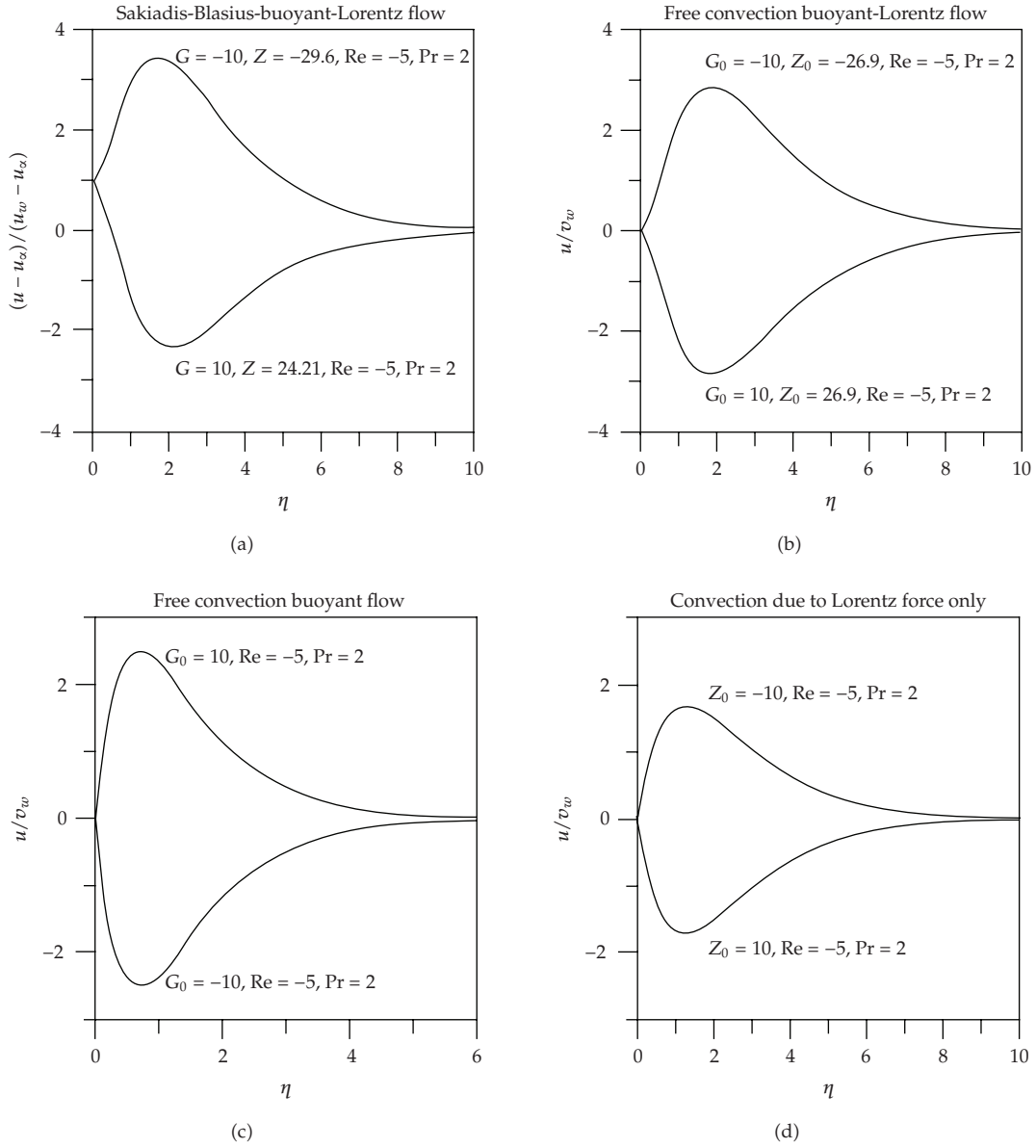
where Re is the suction Reynolds number:

$$\mathrm{Re} = \frac{av_w}{v}. \qquad (3.9)$$

It should be noted here that this is the first work on fluid mechanics which uses both EMHD Lorentz forces and suction simultaneously, and therefore the above Reynolds number, using as characteristic length the distance between the magnets, appears for the first time in the literature.

When

$$Z = -1 - G(1 - \mathrm{Pr})\left(\frac{\pi}{\mathrm{Re}} + 1\right)^{-1}, \qquad (3.10)$$

Figure 3: Dimensionless velocity distribution for different kinds of flow with buoyancy forces and Lorentz forces. In cases (a) and (b), the wall shear stress is zero.

the wall shear stress is zero. In Figure 3(a), two velocity profiles are shown for which the wall shear stress is zero ((3.10) is valid). The transverse coordinate η is

$$\eta = -\frac{v_w}{v}y,$$

(3.11)

and the two curves meet the vertical axis orthogonally.

When the free-stream velocity is zero, we have the Sakiadis-buoyant-EMHD flow:

$$\frac{u}{u_w} = (1 + G_s + Z_s) \exp\left(\frac{v_w}{v}y\right) - G_s \exp\left(\frac{v_w}{\lambda}y\right) - Z_s \exp\left(-\frac{\pi}{a}y\right), \qquad (3.12)$$

where

$$G_s = \frac{\lambda g \beta (T_w - T_\infty)}{u_w v_w^2 (\mathrm{Pr} - 1)},$$

$$Z_s = \frac{j_0 M_0 a^2}{8 \rho_\infty u_w (\pi v + a v_w)}. \qquad (3.13)$$

The absolute wall shear stress is

$$\tau_w = \rho_\infty u_w v_w - \frac{\mu g \beta (T_w - T_\infty)}{v_w \, \mathrm{Pr}} + \frac{j_0 M_0 a}{8}, \qquad (3.14)$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho u_w v_w} = 1 + G_s (1 - \mathrm{Pr}) + Z_s \left(\frac{\pi}{\mathrm{Re}} + 1\right). \qquad (3.15)$$

When

$$Z_s = -1 - G_s (1 - \mathrm{Pr}) \left(\frac{\pi}{\mathrm{Re}} + 1\right)^{-1}, \qquad (3.16)$$

the wall shear stress is zero.

When the plate is motionless, we have the Blasius-buoyant-EMHD flow:

$$\frac{u}{u_\infty} = 1 - (1 - G_b - Z_b) \exp\left(\frac{v_w}{v}y\right) - G_b \exp\left(\frac{v_w}{\lambda}y\right) - Z_b \exp\left(-\frac{\pi}{a}y\right), \qquad (3.17)$$

where

$$G_b = \frac{\lambda g \beta (T_w - T_\infty)}{u_\infty v_w^2 (\mathrm{Pr} - 1)},$$

$$Z_b = \frac{j_0 M_0 a^2}{8 \rho_\infty u_\infty (\pi v + a v_w)}. \qquad (3.18)$$

The absolute wall shear stress is

$$\tau_w = -\rho_\infty u_\infty v_w - \frac{\mu g \beta (T_w - T_\infty)}{v_w \, \mathrm{Pr}} + \frac{j_0 M_0 a}{8}, \qquad (3.19)$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho_\infty u_\infty v_w} = -1 + G_b(1 - \mathrm{Pr}) + Z_b\left(\frac{\pi}{\mathrm{Re}} + 1\right). \tag{3.20}$$

When

$$Z_b = 1 - G_b(1 - \mathrm{Pr})\left(\frac{\pi}{\mathrm{Re}} + 1\right)^{-1}, \tag{3.21}$$

the wall shear stress is zero.

When the plate is motionless and the free-stream velocity is zero, we have pure free convection and the analytical solution becomes

$$\frac{u}{v_w} = (G_0 + Z_0) \exp\left(\frac{v_w}{v}y\right) - G_0 \exp\left(\frac{v_w}{\lambda}y\right) - Z_0 \exp\left(-\frac{\pi}{a}y\right), \tag{3.22}$$

where

$$G_0 = \frac{\lambda g \beta (T_w - T_\infty)}{v_w^3 (\mathrm{Pr} - 1)},$$

$$Z_0 = \frac{j_0 M_0 a^2}{8\rho_\infty v_w(\pi v + a v_w)}. \tag{3.23}$$

In (3.22), the longitudinal velocity u is nondimensionalized with suction velocity v_w because in the above problem the suction velocity is the apparent characteristic velocity of the flow.

The absolute wall shear stress is

$$\tau_w = -\frac{\mu g \beta (T_w - T_\infty)}{v_w \mathrm{Pr}} + \frac{j_0 M_0 a}{8}, \tag{3.24}$$
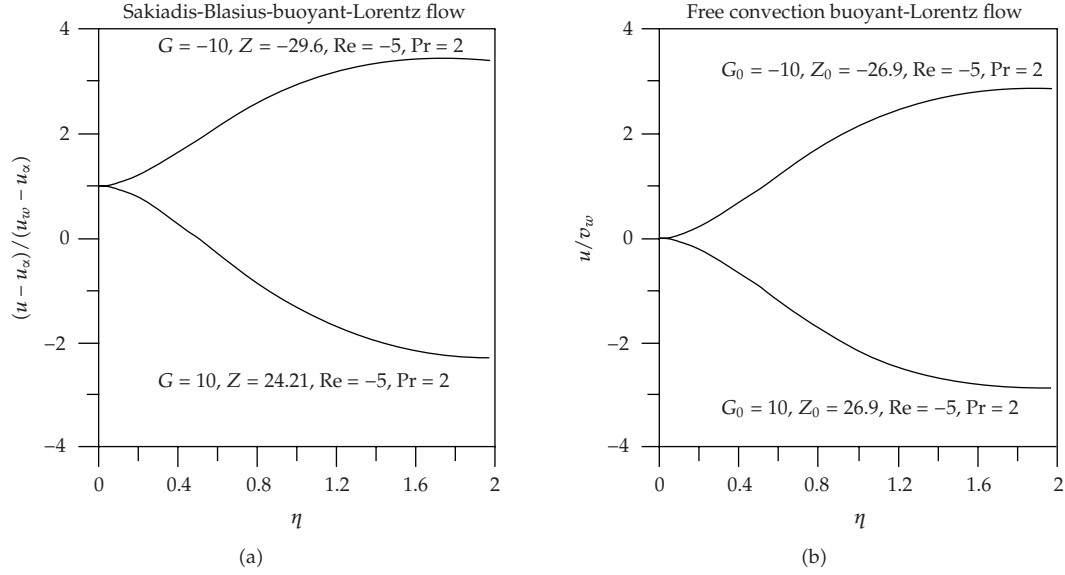
while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho v_w^2} = G_0(1 - \mathrm{Pr}) + Z_0\left(\frac{\pi}{\mathrm{Re}} + 1\right). \tag{3.25}$$

When

$$Z_0 = G_0(\mathrm{Pr} - 1)\left(\frac{\pi}{\mathrm{Re}} + 1\right)^{-1}, \tag{3.26}$$

the wall shear stress is zero. In Figure 3(b), two velocity profiles are shown for which the wall shear stress is zero. The velocity profiles near the plate for cases (a) and (b) are shown in Figure 4.

Figure 4: The velocity profiles near the plate for cases (a) and (b) where it is clearly shown that the wall shear stress is zero.

When there is no Lorentz force, we have convection due to buoyancy force only and velocity is given by

$$\frac{u}{v_w} = G_0 \exp\left(\frac{v_w}{\nu}y\right) - G_0 \exp\left(\frac{v_w}{\lambda}y\right). \tag{3.27}$$

The absolute wall shear stress is

$$\tau_w = -\frac{\mu g \beta (T_w - T_\infty)}{v_w \Pr}, \tag{3.28}$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho v_w^2} = G_0(1 - \Pr). \tag{3.29}$$

When there is no buoyancy force, we have convection due to EMHD Lorentz force only and velocity is given by

$$\frac{u}{v_w} = Z_0 \exp\left(\frac{v_w}{\nu}y\right) - Z_0 \exp\left(-\frac{\pi}{a}y\right). \tag{3.30}$$

This is a new "strange" kind of convection caused by the EMHD Lorentz force only. The physical meaning of the above equation is as follows. Let us suppose that we have a porous plate where fluid is removed with uniform suction. Although the plate is motionless and the

ambient fluid is at rest, a boundary layer forms along the plate due to the act of the Lorentz force which is independent of the flow.

The absolute value of wall shear stress is

$$\tau_w = \frac{j_0 M_0 a}{8},\tag{3.31}$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho v_w^2} = Z_0 \left(\frac{\pi}{\mathrm{Re}} + 1 \right).\tag{3.32}$$

It is surprising that the absolute wall shear stress is independent of viscosity and suction velocity.

3.2. Sakiadis-Blasius flow with buoyancy forces and EMHD Lorentz forces in water near 4°C

The momentum equation with EMHD Lorentz force and the buoyancy force expressed in density is

$$v\frac{\partial^2 u}{\partial y^2} - v_w \frac{\partial u}{\partial y} = -g\frac{\rho_\infty - \rho}{\rho_\infty} - \frac{\pi j_0 M_0}{8\rho_\infty} \exp\left(-\frac{\pi}{a} y \right).\tag{3.33}$$

The buoyancy term can be calculated from the following equation [37]:

$$\frac{\rho_\infty - \rho}{\rho_\infty} = \gamma (T - T_\infty)^2,\tag{3.34}$$

where $\gamma = 0.8 \times 10^{-5} (°C)^{-2}$. Equation (3.34) is valid for the temperature range of $0°C \le T \le 8°C$, and $T_\infty = 4°C$ where the variation of water density is nearly parabolic. Therefore, (3.33) becomes

$$v\frac{\partial^2 u}{\partial y^2} - v_w \frac{\partial u}{\partial y} = -g\gamma (T - T_\infty)^2 - \frac{\pi j_0 M_0}{8\rho} \exp\left(-\frac{\pi}{a} y \right).\tag{3.35}$$

Substituting the temperature difference from (2.10) into (3.35), we have

$$v\frac{\partial^2 u}{\partial y^2} - v_w \frac{\partial u}{\partial y} = -g\gamma (T_w - T_\infty)^2 \exp\left(\frac{2v_w}{\lambda} y \right) - \frac{\pi j_0 M_0}{8\rho} \exp\left(-\frac{\pi}{a} y \right).\tag{3.36}$$

The exact solution of this equation is

$$\frac{u - u_\infty}{u_w - u_\infty} = (1 + W + Z) \exp\left(\frac{v_w}{v} y \right) - W \exp\left(\frac{2v_w}{\lambda} y \right) - Z \exp\left(-\frac{\pi}{a} y \right),\tag{3.37}$$

where

$$W = \frac{\lambda g \gamma (T_w - T_\infty)^2}{2(u_w - u_\infty)v_w^2(2\Pr -1)}. \tag{3.38}$$

The absolute wall shear stress is

$$\tau_w = \rho_\infty(u_w - u_\infty)v_w - \frac{\mu g \gamma (T_w - T_\infty)^2}{2v_w \Pr} + \frac{j_0 M_0 a}{8}, \tag{3.39}$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho_\infty(u_w - u_\infty)v_w} = 1 + W(1 - 2\Pr) + Z\left(\frac{\pi}{\mathrm{Re}} + 1\right). \tag{3.40}$$

The wall shear stress becomes zero when

$$Z = -1 + W(2\Pr -1)\left(\frac{\pi}{\mathrm{Re}} + 1\right)^{-1}. \tag{3.41}$$

When the free-stream velocity is zero, we have the Sakiadis-buoyant-EMHD flow:

$$\frac{u}{u_w} = (1 + W_s + Z_s)\exp\left(\frac{v_w}{\nu}y\right) - W_s\exp\left(\frac{2v_w}{\lambda}y\right) - Z_s\exp\left(-\frac{\pi}{a}y\right), \tag{3.42}$$

where

$$W_s = \frac{\lambda g \gamma (T_w - T_\infty)^2}{2u_w v_w^2(2\Pr -1)}. \tag{3.43}$$

The absolute wall shear stress is

$$\tau_w = \rho_\infty u_w v_w - \frac{\mu g \gamma (T_w - T_\infty)^2}{2v_w \Pr} + \frac{j_0 M_0 a}{8}, \tag{3.44}$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho_\infty u_w v_w} = 1 + W_s(1 - 2\Pr) + Z_s\left(\frac{\pi}{\mathrm{Re}} + 1\right). \tag{3.45}$$

The wall shear stress becomes zero when

$$Z_s = -1 + W_s(2\Pr -1)\left(\frac{\pi}{\mathrm{Re}} + 1\right)^{-1}. \tag{3.46}$$

When the plate is motionless, we have the Blasius-buoyant-EMHD flow:

$$\frac{u}{u_\infty} = 1 - (1 - W_b - Z_b)\exp\left(\frac{v_w}{\nu}y\right) - W_b\exp\left(\frac{2v_w}{\lambda}y\right) - Z_b\exp\left(-\frac{\pi}{a}y\right), \qquad (3.47)$$

where

$$W_b = \frac{\lambda g \gamma (T_w - T_\infty)^2}{2u_\infty v_w^2 (2\Pr - 1)}. \qquad (3.48)$$

The absolute wall shear stress is

$$\tau_w = -\rho_\infty u_\infty v_w - \frac{\mu g \gamma (T_w - T_\infty)^2}{2v_w \Pr} + \frac{j_0 M_0 a}{8}, \qquad (3.49)$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho u_\infty v_w} = -1 + W_b(1 - 2\Pr) + Z_b\left(\frac{\pi}{\mathrm{Re}} + 1\right). \qquad (3.50)$$

The wall shear stress becomes zero when

$$Z_b = 1 + W_b(2\Pr - 1)\left(\frac{\pi}{\mathrm{Re}} + 1\right)^{-1}. \qquad (3.51)$$

When the plate is motionless and the free-stream velocity is zero, we have pure free convection and the analytical solution becomes

$$\frac{u}{v_w} = (W_0 + Z_0)\exp\left(\frac{v_w}{\nu}y\right) - W_0\exp\left(\frac{2v_w}{\lambda}y\right) - Z_0\exp\left(-\frac{\pi}{a}y\right), \qquad (3.52)$$

where

$$W_0 = \frac{\lambda g \gamma (T_w - T_\infty)^2}{2v_w^3 (2\Pr - 1)}. \qquad (3.53)$$

The absolute wall shear stress is

$$\tau_w = -\frac{\mu g \gamma (T_w - T_\infty)^2}{2v_w \Pr} + \frac{j_0 M_0 a}{8}, \qquad (3.54)$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho v_w^2} = W_0(1 - 2\Pr) + Z_0\left(\frac{\pi}{\mathrm{Re}} + 1\right). \qquad (3.55)$$

The wall shear stress becomes zero when

$$Z_0 = W_0(2\Pr - 1)\left(\frac{\pi}{\mathrm{Re}} + 1\right)^{-1}. \tag{3.56}$$

When there is no EMHD Lorentz force, we have convection due to buoyancy force only and velocity is given by

$$\frac{u}{v_w} = W_0 \exp\left(\frac{v_w}{\nu}y\right) - W_0 \exp\left(\frac{2v_w}{\lambda}y\right). \tag{3.57}$$

The absolute wall shear stress is

$$\tau_w = -\frac{\mu g \gamma (T_w - T_\infty)^2}{2 v_w \Pr}, \tag{3.58}$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho v_w^2} = W_0(1 - 2\Pr). \tag{3.59}$$

3.3. Magnetohydrodynamic free convection with uniform suction

The momentum equation for this problem is [24]

$$\nu \frac{\partial^2 u}{\partial y^2} - v_w \frac{\partial u}{\partial y} - \frac{\sigma B^2}{\rho_\infty} u = -g\beta(T_w - T_\infty) \exp\left(\frac{v_w}{\lambda}y\right), \tag{3.60}$$

where σ is the fluid electrical conductivity and B is the strength of magnetic field which is applied transversely to the flow (see Figure 1). The MHD Lorentz force $\sigma B^2 u / \rho_\infty$ always opposes the flow.

The analytical solution of the Sakiadis-buoyant-MHD flow for a moving plate has been produced by Vajravelu [38]. Here we will give the analytical solution for a motionless plate and motionless ambient fluid (pure free convection). The exact solution is

$$\frac{u}{v_w} = G_m\left(\exp\left(A\frac{v_w}{\nu}y\right) - \exp\left(\frac{v_w}{\lambda}y\right)\right), \tag{3.61}$$

Where $A = 0.5(1 + (1 + 4M)^{1/2})$, M is the Hartmann number:

$$M = \frac{\sigma B^2 \nu}{\rho_\infty v_w^2}, \tag{3.62}$$

and G_m is the buoyancy parameter:

$$G_m = \frac{\nu g \beta (T_w - T_\infty)}{v_w^3 (\Pr^2 - \Pr - M)}. \tag{3.63}$$

The absolute wall shear stress is

$$\tau_w = \frac{\mu g \beta (T_w - T_\infty)(A - \text{Pr})}{v_w (\text{Pr}^2 - \text{Pr} - M)},$$
(3.64)

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho_\infty v_w^2} = G_m (A - \text{Pr}).$$
(3.65)

3.4. Magnetohydrodynamic Sakiadis flow in water near 4°C

The momentum equation with magnetic field and buoyancy force expressed in density is

$$v \frac{\partial^2 u}{\partial y^2} - v_w \frac{\partial u}{\partial y} - \frac{\sigma B^2}{\rho_\infty} u = -g \frac{\rho_\infty - \rho}{\rho_\infty}.$$
(3.66)

Taking into account (3.34), we have

$$v \frac{\partial^2 u}{\partial y^2} - v_w \frac{\partial u}{\partial y} - \frac{\sigma B^2}{\rho_\infty} u = -g\gamma (T - T_\infty)^2.$$
(3.67)

Substituting the temperature difference from (2.10) into (3.67), we have

$$v \frac{\partial^2 u}{\partial y^2} - v_w \frac{\partial u}{\partial y} - \frac{\sigma B^2}{\rho_\infty} u = -g\gamma (T_w - T_\infty)^2 \exp\left(\frac{2v_w}{\lambda} y\right).$$
(3.68)

Equation (3.68) has the following analytical solution for Sakiadis-buoyant-MHD flow:

$$\frac{u}{u_w} = (1 + G_{sw}) \exp\left(A \frac{v_w}{v} y\right) - G_{sw} \exp\left(\frac{2v_w}{\lambda} y\right),$$
(3.69)

where

$$G_{sw} = \frac{v g \gamma (T_w - T_\infty)^2}{u_w v_w^2 (4\text{Pr}^2 - 2\text{Pr} - M)}.$$
(3.70)

The absolute wall shear stress is

$$\tau_w = \rho_\infty u_w v_w A + \frac{\mu g \gamma (T_w - T_\infty)^2 (A - 2\text{Pr})}{v_w (4\text{Pr}^2 - 2\text{Pr} - M)},$$
(3.71)

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho_\infty u_w v_w} = A + G_{sw} (A - 2\text{Pr}).$$
(3.72)

When the plate is motionless, we have pure free convection and velocity is given by

$$\frac{u}{v_w} = G_{0w} \exp\left(A\frac{v_w}{v}y\right) - G_{0w} \exp\left(\frac{2v_w}{\lambda}y\right), \tag{3.73}$$

where

$$G_{0w} = \frac{vg\gamma(T_w - T_\infty)^2}{v_w^3(4\Pr^2 - 2\Pr - M)}. \tag{3.74}$$

The absolute wall shear stress is

$$\tau_w = \frac{\mu g\gamma(T_w - T_\infty)^2(A - 2\Pr)}{v_w(4\Pr^2 - 2\Pr - M)}, \tag{3.75}$$

while the dimensionless skin-friction coefficient is

$$c_f = \frac{\tau_w}{\rho_\infty v_w^2} = G_{sw}(A - 2\Pr). \tag{3.76}$$

4. Conclusions

In this paper, the boundary layer buoyant flow along a vertical porous plate with uniform suction has been treated. Exact analytical solutions have been found for Blasius, Sakiadis, and combined Blasius-Sakiadis flows with MHD and EMHD Lorentz forces. A series of new dimensionless parameters (G, Z, W, Re) are introduced for the presentation of the results in elegant form. The author believes that the results of the present work will enrich the list with the existing exact solutions of the Navier-Stokes equations and may help the investigation of flow separation control in aeronautics and hydronautics as well as the application of magnetohydrodynamics in industry.

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