

*Research Article*

# Unsteady Solutions in a Third-Grade Fluid Filling the Porous Space

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An analysis is made of the unsteady flow of a third-grade fluid in a porous medium. A modified Darcy's law is considered in the flow modelling. Reduction and solutions are obtained by employing similarity and numerical methods. The effects of pertinent parameters on the flow velocity are studied through graphs.

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## 1. Introduction

Recently, it has been recognised in industrial and technological applications that non-Newtonian fluids are more appropriate than viscous fluids. However, there is no model which can alone predict the behaviour of all non-Newtonian fluids. The governing equations of non-Newtonian fluids are of higher order than the Navier-Stokes equations. Therefore, the adhering boundary conditions are not sufficient, and one needs additional boundary conditions for a unique solution. Excellent critical reviews in this direction have been given by Rajagopal [1, 2], Rajagopal et al. [3], and Rajagopal and Kaloni [4]. Amongst the several non-Newtonian fluid models, much attention has been paid to the simplest subclass of viscoelastic fluids known as the second grade. However, this model is not capable of describing the shear thinning and thickening phenomena for steady flow over a rigid boundary. The third-grade fluid model represents a further, although inconclusive, attempt towards a more comprehensive description of the behaviour of viscoelastic fluids. Also, the flows of such fluids in porous medium are quite prevalent in many engineering fields such as enhanced oil recovery, paper and textile coating, and composite manufacturing processes. Some important studies dealing with the flows of non-Newtonian fluids are made by Rajagopal and Na [5, 6], Rajagopal and

Gupta [7], Hayat et al. [8–11], Hayat and Ali [12], Ariel et al. [13], Hayat and Kara [14], Abdel-Malek et al. [15], Wafo Soh [16] and Chen et al. [17], and Fetecau and Fetecau [18–20].

Recently, Tan and Masuoka [21] analysed the Stokes' first problem for a second-grade fluid in a porous medium. In another paper, Tan and Masuoka [22] studied the Stokes' first problem for an Oldroyd-B fluid in a porous medium. In these investigations, the authors have used the modified Darcy's law.

The main goal of this paper is to determine analytical solutions for an unsteady flow of a third-grade fluid over a moved plate. The relevant problem is formulated using modified Darcy's law of a third-grade fluid. Two types of analytical solutions are presented and discussed. A numerical solution is also presented.

## 2. Problem formulation

Let us introduce a Cartesian coordinate system OXYZ with  $y$ -axis in the upward direction. The third-grade fluid fills the porous space  $y > 0$  and is in contact with an infinite moved plate at  $y = 0$ . For unidirectional flow, the velocity field is

$$\mathbf{V} = (u(y, t), 0, 0), \quad (2.1)$$

where the above definition of velocity automatically satisfies the incompressibility condition. The equation of motion in a porous medium without body forces is

$$\rho \frac{d\mathbf{V}}{dt} = \text{div } \mathbf{T} + \mathbf{r}, \quad (2.2)$$

where  $\rho$  is the fluid density,  $d/dt$  is the material time differentiation,  $\mathbf{T}$  is the Cauchy stress tensor, and  $\mathbf{r}$  is the Darcy's resistance in a porous space. The Cauchy stress tensor of an incompressible third-grade fluid has the form [23]

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_1\mathbf{A}_3 + \beta_2(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1) + \beta_3(\text{tr } \mathbf{A}_1^2)\mathbf{A}_1, \quad (2.3)$$

in which  $p$  is the pressure,  $\mathbf{I}$  is the identity tensor,  $\alpha_i$  ( $i = 1, 2$ ) and  $\beta_i$  ( $i = 1 - 3$ ) are the material constants, and  $\mathbf{A}_i$  ( $i = 1 - 3$ ) are the first Rivlin-Ericksen tensors [24] which may be defined through the following equations:

$$\begin{aligned} \mathbf{A}_1 &= (\text{grad } \mathbf{V}) + (\text{grad } \mathbf{V})^T, \\ \mathbf{A}_n &= \frac{d\mathbf{A}_{n-1}}{dt} + \mathbf{A}_{n-1}(\text{grad } \mathbf{V}) + (\text{grad } \mathbf{V})^T \mathbf{A}_{n-1}; \quad n > 1. \end{aligned} \quad (2.4)$$

In studying fluid dynamics, it is assumed that the flow meets the Clausius-Duhem inequality and that the specific Helmholtz free energy of the fluid is a minimum at equilibrium when [25]

$$\begin{aligned} \mu \geq 0, \quad \alpha_1 \geq 0, \quad \beta_1 = \beta_2 = 0, \quad \beta_3 \geq 0, \\ |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}. \end{aligned} \quad (2.5)$$

On the basis of constitutive equation in an Oldroyd-B fluid, the following expression in a porous medium has been proposed [26]:

$$\left(1 + \lambda \frac{\partial}{\partial t}\right) \nabla p = -\frac{\mu\phi}{k} \left(1 + \lambda_r \frac{\partial}{\partial t}\right) \mathbf{V}, \quad (2.6)$$

where  $\lambda$  and  $\lambda_r$  are the relaxation and retardation times, and  $\phi$  and  $k$  are the porosity and permeability of the porous medium, respectively. It should be pointed out that for  $\lambda_r = 0$ , (2.6) reduces to the expression which holds for a Maxwell fluid [26] and when  $\lambda = 0$ , it reduces to that of second-grade fluid [22].

Keeping the analogy of (2.6) with the constitutive equation of an extra stress tensor in an Oldroyd-B fluid, the following expression in the present problem has been suggested:

$$\frac{\partial p}{\partial x} = -\frac{\phi}{k} \left[ \mu + \alpha_1 \frac{\partial}{\partial t} + 2\beta_3 \left( \frac{\partial u}{\partial y} \right)^2 \right] u. \quad (2.7)$$

Since the pressure gradient in (2.7) can also be interpreted as a measure of the flow resistance in the bulk of the porous medium, and  $r_x$  is the measure of the flow resistance offered by the solid matrix in  $x$ -direction, then

$$r_x = -\frac{\phi}{k} \left[ \mu + \alpha_1 \frac{\partial}{\partial t} + 2\beta_3 \left( \frac{\partial u}{\partial y} \right)^2 \right] u. \quad (2.8)$$

From (2.1) to (2.5) and (2.8), we have

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + 6\beta_3 \left( \frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} - \left[ \mu + \alpha_1 \frac{\partial}{\partial t} + 2\beta_3 \left( \frac{\partial u}{\partial y} \right)^2 \right] \frac{\phi u}{k}. \quad (2.9)$$

The relevant boundary and initial conditions are

$$u(0, t) = u_0 V(t), \quad t > 0, \quad (2.10)$$

$$u(\infty, t) = 0, \quad t > 0, \quad (2.11)$$

$$u(y, 0) = g(y), \quad y > 0, \quad (2.12)$$

in which  $u_0$  is the reference velocity.

### 3. Solutions of the problem

We rewrite (2.9) as

$$\frac{\partial u}{\partial t} = \mu_* \frac{\partial^2 u}{\partial y^2} + \alpha \frac{\partial^3 u}{\partial y^2 \partial t} + \gamma_1 \left( \frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} - \gamma_2 u \left( \frac{\partial u}{\partial y} \right)^2 - \phi_1 u, \quad (3.1)$$

where

$$\begin{aligned} \mu_* &= \frac{\mu}{\rho + \alpha_1(\phi/k)}, & \alpha &= \frac{\alpha_1}{\rho + \alpha_1(\phi/k)}, & \gamma_1 &= \frac{6\beta_3}{\rho + \alpha_1(\phi/k)}, \\ \gamma_2 &= \frac{2\beta_3(\phi/k)}{\rho + \alpha_1(\phi/k)}, & \phi_1 &= \frac{\mu(\phi/k)}{\rho + \alpha_1(\phi/k)}. \end{aligned} \quad (3.2)$$

### 3.1. Lie symmetry analysis

The Lie symmetry analysis reveals that (3.1) admits two sets of symmetry generators depending on the value of  $\phi_1$ . The appendix provides details of the symmetry analysis of (3.1). *Case 1* ( $\phi_1 \neq \mu_*/\alpha$ ). We obtain a two-dimensional Lie algebra generated by

$$X_1 = \frac{\partial}{\partial y}, \quad X_2 = \frac{\partial}{\partial t}. \quad (3.3)$$

*Case 2* ( $\phi_1 = \mu_*/\alpha$ ). We find a three-dimensional Lie algebra generated by

$$X_1 = \frac{\partial}{\partial y}, \quad X_2 = \frac{\partial}{\partial t}, \quad X_3 = e^{(2\mu_*/\alpha)t} \frac{\partial}{\partial t} - \frac{\mu_*}{\alpha} e^{(2\mu_*/\alpha)t} u \frac{\partial}{\partial u}. \quad (3.4)$$

### 3.2. Travelling wave solutions

We now look for invariant solutions under the operator  $X_1 - cX_2$ , which represents wave-front-type travelling wave solutions with constant wave speed  $c$ . The invariant is given by

$$u(y, t) = U(x_1), \quad \text{where } x_1 = y + ct. \quad (3.5)$$

Substituting (3.5) into (3.1) yields a third-order ordinary differential equation for  $U(x_1)$ ,

$$\phi_1 U(x_1) = -c \frac{dU}{dx_1} - \gamma_2 U(x_1) \left( \frac{dU}{dx_1} \right)^2 + \mu \frac{d^2 U}{dx_1^2} + \gamma_1 \left( \frac{dU}{dx_1} \right)^2 \frac{d^2 U}{dx_1^2} + c\alpha \frac{d^3 U}{dx_1^3}. \quad (3.6)$$

It can be seen that this equation admits the solution

$$U(x_1) = u_0 \exp \left( \frac{\sqrt{\gamma_2} x_1}{-\sqrt{\gamma_1}} \right) \quad (3.7)$$

provided that

$$\frac{\gamma_2}{\gamma_1} \left( \mu - c\alpha \sqrt{\frac{\gamma_2}{\gamma_1}} \right) + \left( c\alpha \sqrt{\frac{\gamma_2}{\gamma_1}} - \phi_1 \right) = 0, \quad (3.8)$$

and hence (3.1) subject to (2.10)–(2.12) admits the solution

$$u(y, t) = u_0 \exp \left( \frac{\sqrt{\gamma_2} (y + ct)}{-\sqrt{\gamma_1}} \right). \quad (3.9)$$

This solution is plotted in Figures 1 and 4 for various values of the emerging parameters.

On the other hand, we find group-invariant solutions corresponding to operators which give meaningful physical solutions of the initial and boundary value problems (2.9) to (2.12). This means  $X_2$  and  $X_3$ .

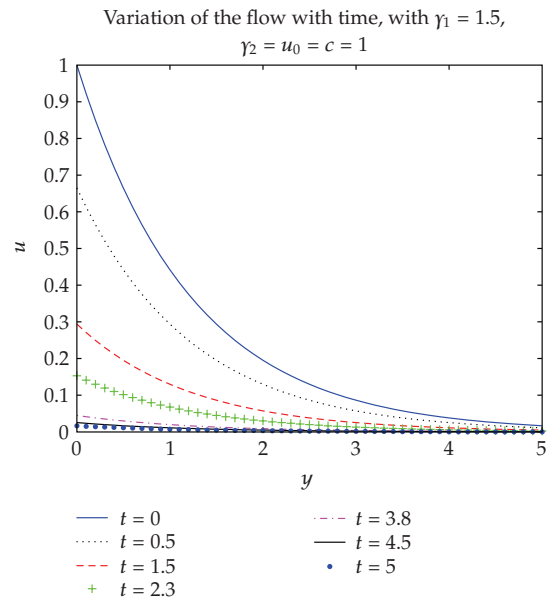
### 3.3. Group-invariant solutions corresponding to $X_2$

The invariant solution admitted by  $X_2$  is the steady-state solution

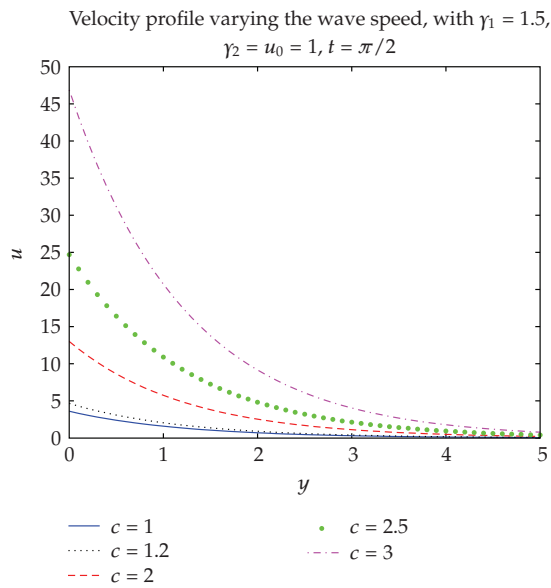
$$u(y, t) = F(y). \quad (3.10)$$

The substitution of (3.10) into (3.1) yields the second-order ordinary differential equation for  $F(y)$ :

$$\gamma_1 (F'(y))^2 F''(y) + \mu_* F''(y) - \gamma_2 F(y) (F'(y))^2 - \phi_1 F(y) = 0, \quad (3.11)$$



**Figure 1:** Travelling wave solutions varying  $t$ .

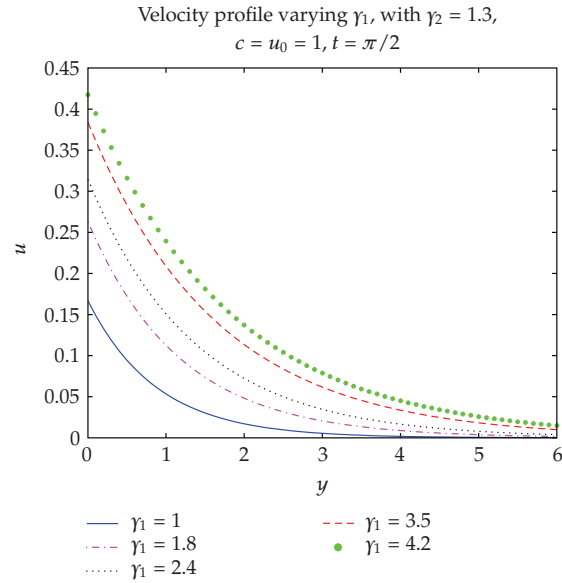


**Figure 2:** Travelling wave solutions varying  $c$ .

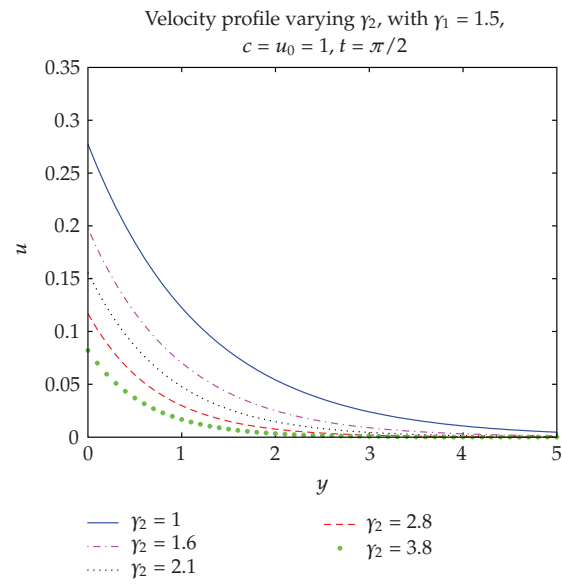
subject to boundary conditions

$$F(0) = v_0, \tag{3.12}$$

$$F(l) = 0, \quad l > 0, \tag{3.13}$$



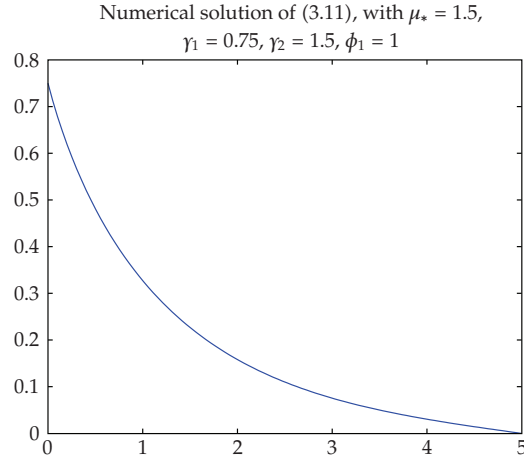
**Figure 3:** Travelling wave solutions varying  $\gamma_1$ .



**Figure 4:** Travelling wave solutions varying  $\gamma_2$ .

where  $l$  is sufficiently large, and  $u_0 V = v_0$  is a constant ( $V$  is taken to be a constant). Let

$$K(F) = \frac{dF}{dy}, \quad (3.14)$$



**Figure 5:** Numerical solution of (3.11) or (3.17) subject to the boundary conditions (3.12) and (3.13).

then (3.11) transforms to

$$\gamma_1 K(F)^3 K'(F) + \mu_* K(F) K'(F) - \gamma_2 F K(F)^2 - \phi_1 F = 0. \quad (3.15)$$

The integration of (3.15) gives

$$\frac{\gamma_1}{\gamma_2} K(F) + \frac{\mu_* \gamma_2 - \gamma_1 \phi_1}{\gamma_2 \sqrt{\gamma_2 \phi_1}}, \quad \text{Arctan} \left[ \sqrt{\frac{\gamma_2}{\gamma_1}} K(F) \right] = \frac{1}{2} y^2 + C, \quad (3.16)$$

where  $C$  is a constant. Equation (3.16) is equivalent to the following first-order ODE in  $F$ :

$$\frac{\gamma_1}{\gamma_2} F'(y) + \frac{\mu_* \gamma_2 - \gamma_1 \phi_1}{\gamma_2 \sqrt{\gamma_2 \phi_1}}, \quad \text{Arctan} \left[ \sqrt{\frac{\gamma_2}{\gamma_1}} F'(x) \right] = \frac{1}{2} y^2 + C. \quad (3.17)$$

One can solve this numerically subject to the boundary conditions (3.12) and (3.13). This solution is plotted in Figure 5.

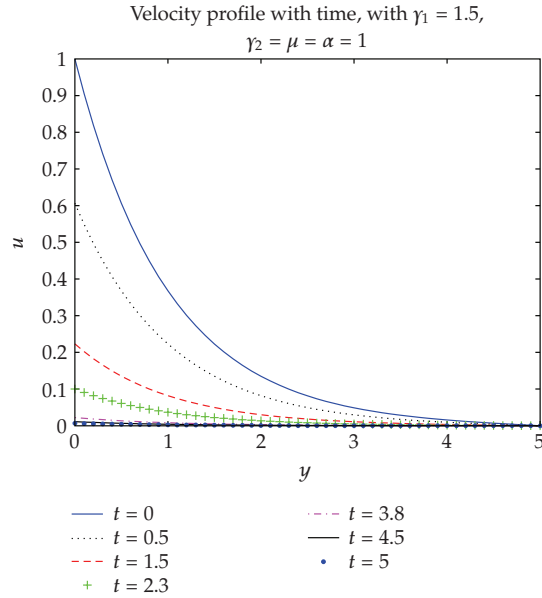
### 3.4. Group-invariant solutions corresponding to $X_3$

The invariant solution admitted by  $X_3$  is

$$u = u_0 \exp\left(\frac{-\mu_*}{\alpha} t\right) B(y), \quad (3.18)$$

where  $B(y)$  as yet is an undetermined function of  $y$ . Substituting (3.18) into (3.1) yields the linear second-order ordinary differential equation

$$B'' - \frac{\gamma_2}{\gamma_1} B = 0. \quad (3.19)$$



**Figure 6:** Analytical solutions for  $\mu_* = 1$ ,  $u_0 = 1$ .

From (2.10) to (2.11), the appropriate boundary conditions for (3.19) are

$$B(0) = 1, \quad B(l) = 0, \quad l \rightarrow \infty, \quad (3.20)$$

where

$$V(t) = u_0 \exp\left(\frac{-\mu_* t}{\alpha}\right). \quad (3.21)$$

We solve (3.19) subject to the boundary conditions (3.20) for positive  $\gamma_2/\gamma_1$  and obtain

$$B = \exp\left(-\sqrt{\frac{\gamma_2}{\gamma_1}} y\right). \quad (3.22)$$

The solutions (3.18) are plotted for positive  $\gamma_2/\gamma_1$  in Figure 6. This solution is similar to (3.9) except that we do not have a condition like (3.8) here.

### 3.5. Numerical solution

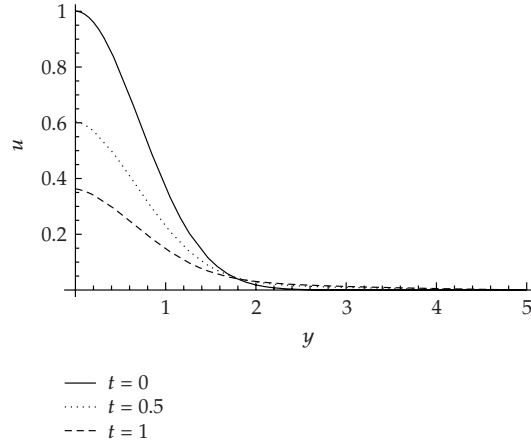
We present the numerical solution of (3.1) subject to the initial and boundary conditions:

$$\begin{aligned} u(0, t) &= u_0 V(t), \quad u(\infty, t) = 0, \quad t > 0, \\ u(y, 0) &= g(y), \quad y > 0, \end{aligned} \quad (3.23)$$

where  $g(y)$  is an arbitrary function of  $y$ .

This solution is plotted using Mathematica's solver *NDSolve*.





**Figure 7:** Numerical solution of (3.1), with  $V(t) = e^{-t}$ ,  $g(y) = e^{-y^2}$ ,  $\mu_* = 2.5$ ,  $u_0 = 1$ ,  $\alpha = 2$ ,  $\gamma_1 = 1.5$ ,  $\gamma_2 = 2.6$ ,  $\phi_1 = 0.8$ .

#### 4. Results and discussion

In order to see the variation of various physical parameters on the velocity, Figures 1–7 have been plotted.

The effect of unsteadiness on the velocity profile is shown in Figure 1. This figure depicts that velocity decreases for large values of time. Clearly, the variation of velocity is observed for  $0 \leq t < 3.8$ . For  $t \geq 3.8$ , the velocity profile remains the same. In other words, one can say that steady-state behaviour is achieved for  $t \geq 3.8$ .

The influence of the wave speed  $c$  on the velocity profile has been presented in Figure 2. It is revealed that velocity decreases by increasing  $c$ . Moreover, the effects of the fluid parameters  $\gamma_1$  and  $\gamma_2$  are given in Figures 3 and 4, respectively. These figures depict that  $\gamma_1$  and  $\gamma_2$  have opposite roles on the velocity. These figures show that velocity increases for large values of  $\gamma_1$  whereas it decreases for increasing  $\gamma_2$ . In Figure 5, the steady-state solution is plotted, and the velocity profile is the same as observed in the case of travelling wave solution.

Further, the analytical solutions (3.19) for  $\mu_*/\alpha > 0$  is plotted in Figure 6. Here as indicated in Figure 6, the velocity profile decreases for large values of  $t$ . Ultimately when  $t \geq 3.8$ , there is almost no variation in velocity.

Finally in Figure 7, we have plotted numerically the velocity profile for small variations of time, and it is observed that the velocity decreases as time increases, which is the the same observation made previously for the analytical solutions.

#### Appendix

The operator

$$X = \tau(t, y, u) \frac{\partial}{\partial t} + \xi(t, y, u) \frac{\partial}{\partial y} + \eta(t, y, u) \frac{\partial}{\partial u} \quad (\text{A.1})$$

is a generator of Lie point symmetry of (3.1) if

$$X^{[3]}(u_t - \mu_* u_{yy} - \alpha u_{tyy} - \gamma_1 (u_y)^2 u_{yy} + \gamma_2 u (u_y)^2 + \phi_1 u)|_{(14)} = 0, \quad (\text{A.2})$$

where

$$X^{[3]} = X + \eta^t \frac{\partial}{\partial u_t} + \eta^y \frac{\partial}{\partial u_y} + \eta^{yy} \frac{\partial}{\partial u_{yy}} + \eta^{tyy} \frac{\partial}{\partial u_{tyy}} \quad (\text{A.3})$$

in which

$$\begin{aligned} \eta^t &= D_t \eta - u_t D_t \tau - u_y D_t \xi, \\ \eta^y &= D_y \eta - u_t D_y \tau - u_y D_y \xi, \\ \eta^{yy} &= D_y \eta^y - u_{ty} D_y \tau - u_{yy} D_y \xi, \\ \eta^{tyy} &= D_t \eta^{yy} - u_{tyy} D_t \tau - u_{yyy} D_t \xi \end{aligned} \quad (\text{A.4})$$

and the total derivative operators are

$$\begin{aligned} D_t &= \frac{\partial}{\partial t} + u_t \frac{\partial}{\partial u} + u_{tt} \frac{\partial}{\partial u_t} + \dots, \\ D_y &= \frac{\partial}{\partial y} + u_y \frac{\partial}{\partial u} + u_{yy} \frac{\partial}{\partial u_y} + \dots. \end{aligned} \quad (\text{A.5})$$

Substituting the expansion of (A.4) into the symmetry condition (A.2) and separating them by powers of the derivatives of  $u$ , since  $\tau$ ,  $\xi$ , and  $\eta$  are independent of the derivatives of  $u$ , lead to the overdetermined system of linear partial differential equations (note that  $\gamma_1$  and  $\gamma_2$  are not zero):

$$\begin{aligned} \tau_y &= \tau_u = 0, \\ \xi_t &= \xi_y = \xi_u = 0, \\ \eta_x &= \eta_{uu} = 0, \\ \tau_t \mu_* + \alpha \eta_{tu} &= 0, \\ u \eta_u + \tau_t u + \eta &= 0, \\ \eta_t + \phi_1 \eta + \phi_1 u (\tau_t - \eta_u) &= 0. \end{aligned} \quad (\text{A.6})$$

The solution of this linear system (A.6) gives rise to two cases  $\phi_1 \neq \mu_*/\alpha$  and  $\phi_1 = \mu_*/\alpha$ . In the former, we obtain

$$\xi = a_1, \quad \tau = a_2, \quad \eta = 0, \quad (\text{A.7})$$

and for the second case

$$\xi = a_1, \quad \tau = -\frac{a_2}{\phi_1} \exp(2\phi_1 t) + a_3, \quad \eta = a_2 u \exp(2\phi_1 t). \quad (\text{A.8})$$

In both (A.7) and (A.8), the  $a_i$  are constants. Setting one of the constants  $a_i$  equal to one and the rest of the constants to zero results in the generators given in Section 3.1.

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## Special Issue on Modeling Experimental Nonlinear Dynamics and Chaotic Scenarios

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