Comparison Projection Method with Adomian's Decomposition Method for Solving System of Integral Equations

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Abstract: System of integral equations has been solved in many papers, especially, system of integral equations with degenerate kernels has been solved with Adomian's decomposition method by some authors. In present paper, we try to solve system of integral equations by using collocation method with Legendre polynomials which is more efficient and needs less computations than Adomian's decomposition method.

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1 Introduction

Consider system of Fredholm integral equations [18]

$$\lambda \mathbf{u}(x) = \mathbf{f}(x) + \int_a^b \mathbf{k}(x,t)\mathbf{u}(t)dt, \tag{1}$$

where $\lambda \in \mathbb{R}$, and

$$\begin{aligned} \mathbf{u}(x) &= [u_i(x)], \qquad i = 1, ..., n, \\ \mathbf{f}(x) &= [f_i(x)], \qquad i = 1, ..., n, \\ \mathbf{k}(x,t) &= [k_{i,j}(x,t)], \quad i,j = 1, ..., n. \end{aligned}$$

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This type of equations has been solved in many papers with different methods such as Taylor's expansion [11, 13], operational matrices method [4, 15], homotopy perturbation method [1, 9], Sinc collocation method [16, 17] and Adomian's decomposition method [5, 8, 12]. All of these methods involve solving this kind of equations, but some of them have restrictions such as $k_{i,j}(x,t)$ being degenerate and some of them are focused on system of integral equations of the second kind, with more computations leading to solution with low accuracy. The aim of this paper is to solve system of integral equations by using collocation method with Legendre polynomials [2] as the basis for this projection method, and compare this method with Adomian decomposition method which has been used for solving this type of equations in [7]. Convergence of Legendre polynomials for solving Fredholm integral equation of the second kind has been discussed in [14], which we shall apply this discussion for system of integral equations.

2 Discretization of equations

In this section we apply collocation method to convert equation (1) to algebraic system of linear equations $AX = b$. For this result, by using Legendre polynomials, we approximate $u_i(x)$'s, such that

$$u_i(x) \cong \sum_{k=1}^{m} c_{ik} L_{k-1}(x), \tag{2}$$

where $L_k(x)$ is kth Legendre polynomial and c_{ik}'s are unknown coefficients which are determined by solving an algebraic system of linear equations $AX = b$. By substituting relation (2) in (1) we have

$$\begin{aligned}
\lambda \sum_{k=1}^{m} c_{1k} L_{k-1}(x) &= f_1(x) + \sum_{i=1}^{n} \int_a^b k_{1i}(x,t) \sum_{k=1}^{m} c_{ik} L_{k-1}(t)dt, \\
\lambda \sum_{k=1}^{m} c_{2k} L_{k-1}(x) &= f_2(x) + \sum_{i=1}^{n} \int_a^b k_{2i}(x,t) \sum_{k=1}^{m} c_{ik} L_{k-1}(t)dt, \\
&\vdots \\
\lambda \sum_{k=1}^{m} c_{nk} L_{k-1}(x) &= f_n(x) + \sum_{i=1}^{n} \int_a^b k_{ni}(x,t) \sum_{k=1}^{m} c_{ik} L_{k-1}(t)dt.
\end{aligned}$$

Now, we choose some collocation points such as

$$x_i = a + \frac{i(b-a)}{m}, \qquad i = 1, 2, \ldots, m,$$

which are equidistant, and define system of residual equations by

$$R_1(x) = \lambda \sum_{k=1}^{m} c_{1k} L_{k-1}(x) - f_1(x) - \sum_{i=1}^{n} \int_a^b k_{1i}(x,t) \sum_{k=1}^{m} c_{ik} L_{k-1}(t)dt,$$

$$R_2(x) = \lambda \sum_{k=1}^{m} c_{2k} L_{k-1}(x) - f_2(x) - \sum_{i=1}^{n} \int_a^b k_{2i}(x,t) \sum_{k=1}^{m} c_{ik} L_{k-1}(t) dt,$$

$$\vdots$$

$$R_n(x) = \lambda \sum_{k=1}^{m} c_{nk} L_{k-1}(x) - f_n(x) - \sum_{i=1}^{n} \int_a^b k_{ni}(x,t) \sum_{k=1}^{m} c_{ik} L_{k-1}(t) dt.$$

Then, by imposing the conditions

$$R_i(x_j) = 0, \qquad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m,$$

(where x_j's are collocation points) to system of residual equations, we deduce an algebraic system of linear equations $AX = b$ [3, 10].
For example, for $n = 2$ we have

$$\begin{cases} \lambda u_1(x) = f_1(x) + \int_a^b k_{11}(x,t) u_1(t) dt + \int_a^b k_{12}(x,t) u_2(t) dt, \\ \\ \lambda u_2(x) = f_2(x) + \int_a^b k_{21}(x,t) u_1(t) dt + \int_a^b k_{22}(x,t) u_2(t) dt, \end{cases} \tag{3}$$

which after discretization, an algebraic system of linear equations $AX = b$ is derived as follows

$$A = (a_{ij}), \qquad i, j = 1, 2, \ldots, 2m,$$
$$b^T = [f_1(x_1), f_1(x_2), \ldots, f_1(x_m), f_2(x_1), f_2(x_2), \ldots, f_2(x_m)],$$
$$X^T = [c_{11}, c_{12}, \ldots, c_{1m}, c_{21}, c_{22}, \ldots, c_{2m}],$$

where

$$a_{ij} = \begin{cases} \lambda L_{j-1}(x_i) - \int_a^b k_{11}(x_i,t) L_{j-1}(t) dt, & \begin{cases} i = 1, 2, \ldots, m \\ j = 1, 2, \ldots, m \end{cases} \\ \\ -\int_a^b k_{12}(x_i,t) L_{j-m-1}(t) dt, & \begin{cases} i = 1, 2, \ldots, m \\ j = m+1, \ldots, 2m \end{cases} \\ \\ -\int_a^b k_{21}(x_{i-m},t) L_{j-1}(t) dt, & \begin{cases} i = m+1, \ldots, 2m \\ j = 1, 2, \ldots, m \end{cases} \\ \\ \lambda L_{j-m-1}(x_{i-m}) - \int_a^b k_{22}(x_{i-m},t) L_{j-m-1}(t) dt, & \begin{cases} i = m+1, \ldots, 2m \\ j = m+1, \ldots, 2m. \end{cases} \end{cases}$$

3 Convergence of Method

In this section by using the following Proposition we try to prove a convergence theorem which shows the error bound of the numerical method that we applied in the previous section.

Proposition 1. *Let $f(t) \in H^k(-1,1)$ Sobolev space, $P_m(f(t)) = \sum_{i=0}^{m} a_i L_i(t)$ be the best approximation polynomial of $f(t)$ in L_2-norm. Then*

$$\|f(t) - P_m(f(t))\|_{L_2[-1,1]} \leq C_0 m^{-k} \|f(t)\|_{H^k(-1,1)},$$

where C_0 is a positive constant, which depends on the selected norm and is independent of $f(t)$ and m.

See [6], for the proof of Proposition 1.
For the above proposition we have defined the following norms

$$\|f(t)\|_{L_2[-1,1]} = \left(\int_{-1}^{1} f^2(t) dt \right)^{1/2},$$

$$\|f(t)\|_{H^k(-1,1)} = \left(\sum_{i=0}^{k} \int_{-1}^{1} |f^{(i)}(t)|^2 dt \right)^{1/2}.$$

Theorem 1. *Assume $\kappa : H^k(-1,1) \to L_2[-1,1]$ is an operator defined by*

$$\kappa(\boldsymbol{u}(x)) = \int_{-1}^{1} \boldsymbol{k}(x,t) \boldsymbol{u}(t) dt,$$

where $\boldsymbol{k}(x,t) \in L_2$ in square $[-1,1] \times [-1,1]$ which was introduced in equation 1, and $\mathbf{u}_m(x)$ is the numerical solution of the equation 1. Then

$$\sup_{x \in [-1,1]} |\boldsymbol{u}(x) - \boldsymbol{u}_m(x)| \leq C_1 m^{-k} \|\boldsymbol{u}(t)\|_{H^k(-1,1)},$$

where C_1 is a positive constant.

Proof. Assume that the exact solution of equation 1 is $\mathbf{u}(x)$, i.e.

$$\mathbf{u}(x) = \mathbf{f}(x) + \int_{-1}^{1} \mathbf{k}(x,t) \mathbf{u}(t) dt.$$

If we define the numerical solution of this equation by $\mathbf{u}_m(x)$, then

$$\mathbf{u}_m(x) = \mathbf{f}(x) + \int_{-1}^{1} \mathbf{k}(x,t) P_m(\mathbf{u}(t)) dt.$$

Hence

$$\sup_{x\in[-1,1]} |\mathbf{u}(x) - \mathbf{u}_m(x)| \leq \left| \int_{-1}^{1} \mathbf{k}(x,t)\mathbf{u}(t)dt - \int_{-1}^{1} \mathbf{k}(x,t)P_m(\mathbf{u}(t))dt \right|$$

$$\leq \left(\int_{-1}^{1} \mathbf{k}^2(x,t)dt \right)^{1/2} \|\mathbf{u} - P_m(\mathbf{u})\|.$$

Since $\mathbf{k}(x,t) \in L_2$,

$$\max_{x\in[-1,1]} \left(\int_{-1}^{1} \mathbf{k}^2(x,t)dt \right)^{1/2} \leq M.$$

By using Proposition 1, we have

$$\|\mathbf{u} - P_m(\mathbf{u})\| \leq C_0 m^{-k} \|\mathbf{u}(t)\|_{H^k(-1,1)},$$

and finally,

$$\sup_{x\in[-1,1]} |\mathbf{u}(x) - \mathbf{u}_m(x)| \leq M C_0 m^{-k} \|\mathbf{u}(t)\|_{H^k(-1,1)}.$$

Letting $C_1 = MC_0$ completes proof of the theorem. □

4 Numerical Experiments

In this section, we compare Adomian's decomposition method which has been stated in [7] with Legendre collocation method and present some examples that show the drawbacks of the Adomian's decomposition method.
In [7], Adomian's decomposition method was defined, and for solving system of integral equations 1, introduce the following successive process

$$\mathbf{u}_{n+1} = \mathbf{G}\mathbf{a}^{(n)} = \mathbf{G}\mathbf{B}\mathbf{a}^{(n-1)},$$

where $\mathbf{B}$, $\mathbf{G}$ and $\mathbf{a}$ were defined in [7]. Then the solution of (1) is given by $\mathbf{u} = \sum_{i=0}^{L} \mathbf{u}_i$. In this process, if we increase the number of iterations, high powers of matrix $\mathbf{B}$ are needed to compute. Also round-off errors in computing powers of $\mathbf{B}$ and $\mathbf{a}^{(0)}$ are other weak points of this process that destroy the accuracy of solution. In Adomian's decomposition method we need to generate matrix $\mathbf{B}$ which depends on the number of terms of degenerate kernels, for example if each $k_{ij};\ \ i,j = 1,2$ has only one term then the rank of matrix $\mathbf{B}$ will be (4×4), by increasing the terms of kernels to two, the rank of matrix will be (8×8). So, the entries of matrix $\mathbf{B}$ will increase exponentially which lead to so much computations.

Example 1. In [7], the system of Fredholm integral equations

$$\begin{cases} u_1(x) = \frac{2}{3}e^x - \frac{1}{4} + \int_0^1 (\frac{1}{3}e^x t u_1(t) + t^2 u_2(t))dt, \\ u_2(x) = \frac{3}{2}x - x^2 + \int_0^1 (x^2 e^{-t} u_1(t) - x u_2(t))dt, \end{cases}$$

Table 1: Numerical results for Example 1

	$u_1(x)$		$u_2(x)$	
x	$E_{Adomian}$	$E_{Legendre}$	$E_{Adomian}$	$E_{Legendre}$
0.0	2×10^{-7}	1.66392×10^{-10}	0	2.0212×10^{-14}
0.25	2.56805×10^{-7}	1.23479×10^{-12}	3.75×10^{-8}	3.53051×10^{-14}
0.5	3.29744×10^{-7}	1.02318×10^{-12}	5×10^{-8}	7.01772×10^{-13}
0.75	4.234×10^{-7}	1.0103×10^{-12}	3.75×10^{-8}	1.99962×10^{-12}
1	5.43656×10^{-7}	1.21414×10^{-12}	1.11022×10^{-16}	3.92941×10^{-12}

has been solved by Adomian's decomposition method, where the exact solutions are $u_1(x) = e^x$ and $u_2(x) = x$. After 30 steps of this method which need lots of computations, the following solutions are obtained

$$\begin{cases} u_1(x) = 1.0000002\, e^x, \\ u_2(x) = 1.0000002\, x - 0.0000002\, x^2. \end{cases}$$

We have solved this system by using Legendre collocation method which was defined in previous section for $m = 10$. The numerical results are shown in Table 1. In this table $E_{Adomian}$ and $E_{Legendre}$ mean errors of Adomian's decomposition method and Legendre collocation method for different values of x, respectively.

Example 2. In this example which has been stated in [7], we have the following system

$$\begin{cases} u_1(x) = \frac{1}{18}x + \frac{17}{36} + \int_0^1 \frac{x+t}{3}(u_1(t) + u_2(t))dt, \\ \\ u_2(x) = x^2 - \frac{19}{12}x + 1 + \int_0^1 xt(u_1(t) + u_2(t))dt, \end{cases}$$

where the exact solutions are $u_1(x) = 1 + x$ and $u_2(x) = 1 + x^2$. The results from Adomian's decomposition method for 30 steps given in [7] are compared with Legendre collocation method for $m = 10$ in Table 2.

Table 2: Numerical results for Example 2

	$u_1(x)$		$u_2(x)$	
x	$E_{Adomian}$	$E_{Legendre}$	$E_{Adomian}$	$E_{Legendre}$
0.0	4.7×10^{-5}	3.04201×10^{-14}	0	4.66294×10^{-14}
0.25	6×10^{-6}	1.33227×10^{-15}	4×10^{-6}	6.66134×10^{-16}
0.5	8×10^{-6}	1.55431×10^{-15}	7×10^{-6}	4.44089×10^{-16}
0.75	9×10^{-6}	1.55431×10^{-15}	1.1×10^{-5}	1.33227×10^{-15}
1	1.1×10^{-5}	2.22045×10^{-15}	1.5×10^{-5}	1.33227×10^{-15}

Example 3. In this example, we present a system of integral equations with degenerate kernels, but unfortunately the Adomian's decomposition method [7] is not able to solve this system. In regard to the Adomian's decomposition method for the system

$$\begin{cases} u_1(x) = -\frac{47}{30} + 2x + \frac{17}{12}x^2 + x^3 + \int_0^1 (2t^2 - x^2)(u_1(t) + u_2(t))dt, \\ u_2(x) = -\frac{1}{3}x - \frac{121}{60}x^2 + \int_0^1 3x^2 t(u_1(t) + u_2(t))dt, \end{cases}$$

where the exact solutions are $u_1(x) = x^3 + 2x$ and $u_2(x) = x^2 - \frac{x}{3}$ we have the following process for solving it

$$\mathbf{u}(x) = [u_1(x), u_2(x)],\ \mathbf{f}(x) = [f_1(x), f_2(x)] = [-\frac{47}{30} + 2x + \frac{17}{12}x^2 + x^3, -\frac{1}{3}x - \frac{121}{60}x^2],$$

$$g_{11}(x) = 1,\ \ g_{12}(x) = x^2,\ \ g_{13}(x) = 1,\ \ g_{14}(x) = x^2,\ \ g_{21}(x) = x^2,\ \ g_{22}(x) = x^2,$$

$$h_{11}(t) = 2t^2,\ \ h_{12}(t) = -1,\ \ h_{13}(t) = 2t^2,\ \ h_{14}(t) = -1,\ \ h_{21}(t) = 3t,\ \ h_{22}(t) = 3t,$$

$$\begin{aligned} \mathbf{a}^{(0)} &= \int_0^1 [h_{11}f_1,\ h_{12}f_1,\ h_{13}f_2,\ h_{14}f_2,\ h_{21}f_1,\ h_{22}f_2]dt \\ &= [\frac{77}{90}, \frac{-7}{45}, \frac{-73}{75}, \frac{151}{180}, \frac{21}{16}, \frac{-443}{240}] \end{aligned}$$

and the matrices $\mathbf{G}, \mathbf{B}$ are as follows

$$\mathbf{G} = \begin{bmatrix} g_{11}(x) & g_{12}(x) & g_{13}(x) & g_{14}(x) & 0 & 0 \\ 0 & 0 & 0 & 0 & g_{21}(x) & g_{22}(x) \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \int_0^1 h_{11}g_{11}dt & \int_0^1 h_{11}g_{12}dt & \int_0^1 h_{11}g_{13}dt & \int_0^1 h_{11}g_{14}dt & 0 & 0 \\ \int_0^1 h_{12}g_{11}dt & \int_0^1 h_{12}g_{12}dt & \int_0^1 h_{12}g_{13}dt & \int_0^1 h_{12}g_{14}dt & 0 & 0 \\ 0 & 0 & 0 & 0 & \int_0^1 h_{13}g_{21}dt & \int_0^1 h_{13}g_{22}dt \\ 0 & 0 & 0 & 0 & \int_0^1 h_{14}g_{21}dt & \int_0^1 h_{14}g_{22}dt \\ \int_0^1 h_{21}g_{11}dt & \int_0^1 h_{21}g_{12}dt & \int_0^1 h_{21}g_{13}dt & \int_0^1 h_{21}g_{14}dt & 0 & 0 \\ 0 & 0 & 0 & 0 & \int_0^1 h_{22}g_{21}dt & \int_0^1 h_{22}g_{22}dt \end{bmatrix}$$

so that

$$\mathbf{B} = \begin{bmatrix} \frac{2}{3} & \frac{2}{5} & \frac{2}{3} & \frac{2}{5} & 0 & 0 \\ -1 & \frac{-1}{3} & -1 & \frac{-1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{5} & \frac{2}{5} \\ 0 & 0 & 0 & 0 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{3}{2} & \frac{3}{4} & \frac{3}{2} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{3}{4} \end{bmatrix}$$

Using the above information and the fact $\mathbf{a}^{(n)} = \mathbf{B}\mathbf{a}^{(n-1)}$ for 30 steps of [7], we obtain

$$
\begin{aligned}
u_{10} &= -\frac{47}{30} + 2x + \frac{17}{12}x^2 + x^3, & u_{20} &= -\frac{1}{3}x - \frac{121}{60}x^2,\\
u_{11} &= -\frac{53}{450} + \frac{41}{60}x^2, & u_{21} &= -\frac{8}{15}x^2,\\
u_{12} &= -\frac{1}{54} + \frac{61}{900}x^2, & u_{22} &= -\frac{77}{1200}x^2,\\
&\vdots & &\vdots\\
u_{1,30} &= -0.0120628 + 0.0153814x^2, & u_{2,30} &= -0.0256153x^2.
\end{aligned}
$$

The approximated solution for some values of x after 30 steps of [7] and Legendre collocation method by $m = 10$ are given in Table 3 which shows the advantage of the Legendre collocation method.

Table 3: Numerical results for Example 3 by 30 steps

	$u_1(x)$		$u_2(x)$	
x	$E_{Adomian}$	$E_{Legendre}$	$E_{Adomian}$	$E_{Legendre}$
0.0	2.01549	1.45472×10^{-13}	0.0	2.24643×10^{-14}
0.25	1.85487	7.70495×10^{-14}	0.267494	1.10849×10^{-14}
0.5	1.37299	5.66214×10^{-14}	1.06998	4.41869×10^{-14}
0.75	0.569873	2.28706×10^{-14}	2.40745	9.94205×10^{-14}
1	0.554498	2.44249×10^{-14}	4.2799	1.7697×10^{-13}

Increasing the number of iterations provide no improvement in accuracy of solutions. Numerical results for the above system by 60 steps for Adomian's decomposition method are presented in Table 4.

Table 4: Numerical results for Example 3 by 60 steps

x Values	0	0.25	0.5	0.75	1
$E_{Adomian(u_1)}$	2.4132	2.22088	1.64392	0.682323	0.663914
$E_{Adomian(u_2)}$	0.0	0.320277	1.28111	2.8825	5.12444

Example 4. In this example, we try to solve a system of equations which has nonseparable kernels. It is mentioned In [7] that if the kernels in system of integral equations are nonseparable, then Taylor's expansion method can degenerate them and the Adomian's decomposition method [7] is able to solve it. This claim is not correct because of two reasons, first of all, Taylor's expansion has not enough authority to approximate a function with two variables, second reason is that if

we increase the terms of Taylor's expansion, the rank of matrix B will increase exponentially which leads more computations with lots of round off errors.
Now, in equation (3) let

$$\begin{cases} k_{11}(x,t) = \sin(xt+t) \\ k_{12}(x,t) = e^{xt} \\ k_{21}(x,t) = x^2t - t^3 \\ k_{22}(x,t) = e^{\frac{xt^3}{2}} \\ f_1(x) = \frac{2-e^x(2-2x+x^2)}{x^3} + \cos(4\pi x) + \frac{(1+x)(\cos(1+x)-1)}{(1+x)^2-16\pi^2} \\ f_2(x) = \frac{3}{16\pi^2} - \frac{2(e^{\frac{x}{2}}-1)}{3x} + x^2 \end{cases}$$

and the exact solutions are $u_1(x) = \cos(4\pi x)$ and $u_2(x) = x^2$. Numerical results for Legendre collocation method with $m = 15$ and $m = 20$ are shown in Tables 5 and 6.

Table 5: Numerical results for Example 4 for $m = 15$

x Values	0	0.25	0.5	0.75	1
$E_{Legendre(u_1)}$	4.356×10^{-5}	5.342×10^{-5}	6.139×10^{-5}	7.019×10^{-5}	7.830×10^{-5}
$E_{Legendre(u_2)}$	9.505×10^{-6}	1.223×10^{-5}	1.816×10^{-5}	2.742×10^{-5}	4.002×10^{-5}

Table 6: Numerical results for Example 4 for $m = 20$

x Values	0	0.25	0.5	0.75	1
$E_{Legendre(u_1)}$	5.417×10^{-5}	2.445×10^{-7}	2.827×10^{-7}	3.178×10^{-7}	3.612×10^{-7}
$E_{Legendre(u_2)}$	7.026×10^{-8}	6.684×10^{-8}	9.293×10^{-8}	1.334×10^{-7}	1.883×10^{-7}

Now, if we degenerate the kernels of above system and then solve the new system by Legendre collocation method for $m = 20$, the following numerical results are obtained in Table 7 that show the weakness of Taylor's expansion for degenerating of kernels in spite of the good accuracy that we got in Table 6.

Table 7: Numerical results for Example 4 by Taylor's expansion of order 5.

x Values	0	0.25	0.5	0.75	1
E_{u_1}	2.256×10^{-2}	2.822×10^{-2}	3.613×10^{-2}	4.996×10^{-2}	7.438×10^{-2}
E_{u_2}	1.331×10^{-2}	1.046×10^{-2}	2.702×10^{-3}	1.016×10^{-2}	2.833×10^{-2}

In the following we present an example to show the efficiency of this numerical method for solving first kind system of integral equations.

Example 5. In this example, we try to solve a system of equations

$$\begin{cases} f_1(x) = \int_{-1}^{1} \sin(x^2+t)u_1(t)dt - \int_{-1}^{1} 3te^{x^2t^2}u_2(t)dt, \\ f_2(x) = -\int_{-1}^{1} 3\cos(xt)u_1(t)dt + \int_{-1}^{1} txe^{3xt^2}u_2(t)dt, \end{cases}$$

where

$$\begin{cases} f_1(x) = -\frac{3e^{x^2-1}(-e+e^{x^4})}{2(1+x^2)} - \frac{\sin(x^2+3x)+\sin(x^2-3)}{6} + \frac{(\sin(x-x^2)+\sin(x^2+1))}{2}, \\ f_2(x) = \frac{e^{3x^3+x^2}-e^{3x+1}}{2(e+3ex)} + \frac{-6\cos 2x\cos x^2+3x\sin 2\sin x+6\cos x(\cos 2-x\sin x\sin x^2)}{x^2-4}, \end{cases}$$

and the exact solutions are $u_1(x) = \sin 2x$ and $u_2(x) = e^{x^2-1}$. Numerical results for Legendre collocation method with $m = 10$ and $m = 15$ are shown in Table 8.

Table 8: Numerical results for Example 5

	E_{u_1}		E_{u_2}	
x	$m = 10$	$m = 15$	$m = 10$	$m = 15$
-1	1.7036×10^{-7}	4.5999×10^{-10}	1.0224×10^{-5}	5.4499×10^{-8}
-0.75	9.6772×10^{-9}	1.4551×10^{-11}	1.7953×10^{-7}	6.3424×10^{-10}
-0.5	5.2555×10^{-9}	1.4789×10^{-11}	6.9364×10^{-8}	1.7594×10^{-10}
-0.25	3.5251×10^{-9}	1.0557×10^{-11}	9.4624×10^{-10}	1.1806×10^{-10}
0	3.3230×10^{-10}	4.1108×10^{-13}	2.2215×10^{-7}	1.0512×10^{-10}
0.25	2.4917×10^{-8}	7.3872×10^{-11}	1.3333×10^{-8}	8.2249×10^{-11}
0.5	1.7392×10^{-7}	4.3160×10^{-10}	1.0306×10^{-8}	3.3478×10^{-11}
0.75	8.4662×10^{-7}	2.4831×10^{-9}	9.7882×10^{-8}	1.4628×10^{-10}
1	3.8222×10^{-5}	1.1492×10^{-7}	5.8332×10^{-6}	4.3467×10^{-8}

Conclusion

In this paper, a projection method known as collocation method with Legendre polynomials are chosen to discretize the system of integral equations, this method has some advantages. This method is easy to apply for first and second kind system of integral equations. It also requires less computations than other methods discussed in [7, 9, 11, 15, 16, 17]. For example when this method is applied to many systems of integral equations, by solving an algebraic system of linear equations with rank less than 10×10, we can get good accuracy. In some methods the kernels of system are required to satisfy some conditions such as being separable but the method of this paper does not have such conditions.

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