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Differential Subordination and Superordination For Analytic Functions Defined Using A Family Of Generalized Differential Operators

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Abstract

By making use of the generalized differential operator, the authors derive the subordination and superordination results for certain normalized analytic functions in the open unit disk. Many of the well-known and new results are shown to follow as special cases of our results.

1 Preliminaries

Let $\mathcal{H}$ be the class of functions analytic in the open unit disc $\mathcal{U} = \{z : \mid z \mid < 1\}$. Let $\mathcal{H}(a, n)$ be the subclass of $\mathcal{H}$ consisting of functions of the form $f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \ldots$.
Let

$$\mathcal{A}_n = \{f \in \mathcal{H},\ f(z) = z + a_{n+1} z^{n+1} + a_{n+2} z^{n+2} + \ldots\}$$

and let $\mathcal{A} = \mathcal{A}_1$. Let the functions f and g be analytic in $\mathcal{U}$. We say that the function f is subordinate to g if there exists a Schwarz function w, analytic in $\mathcal{U}$ with $w(0) = 0$ and $\mid w(z) \mid < 1$ such that $f(z) = g(w(z))$ for $z \in \mathcal{U}$. We denote it by $f(z) \prec g(z)$. In particular, if the function g is univalent in $\mathcal{U}$, the above subordination is equivalent to $f(0) = g(0)$ and $f(\mathcal{U}) \subset g(\mathcal{U})$. Let $p, h \in \mathcal{H}$

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and let $\phi(r,s,t;z) : \mathbb{C}^3 \times \mathcal{U} \longrightarrow \mathbb{C}$. If p and $\phi(p(z), zp^{'}(z), z^2p^{''}(z); z)$ are univalent and if p satisfies the second order superordination

$$h(z) \prec \phi(p(z), zp^{'}(z), z^2p^{''}(z); z), \tag{1.1}$$

then p is a solution of the differential superordination (1.1). (If f is subordinate to F, then F is called to be superordinate to f) An analytic function q is called a subordinant if $q \prec p$ for all p satisfying (1.1). An univalent subordinant $\hat{q}$ that satisfies $q \prec \hat{q}$ for all subordinants q of (1.1) is said to be the best subordinant. Recently Miller and Mocanu [6] obtained conditions h, q and ϕ for which the following implication holds:

$$h(z) \prec \phi(p(z), zp^{'}(z), z^2p^{''}(z); z) \implies q(z) \prec p(z).$$

With the results of Miller and Mocanu [6], Bulboacă [3] investigated certain classes of first order differential superordinations as well as superordination-preserving integral operators [4]. Ali et al.[2] used the results obtained by Bulboacă [4] and gave the sufficient conditions for certain normalized analytic functions f to satisfy

$$q_1(z) \prec \frac{zf^{'}(z)}{f(z)} \prec q_2(z)$$

where q_1 and q_2 are given univalent functions in $\mathcal{U}$ with $q_1(0) = 1$ and $q_2(0) = 1$. Shanmugam et al. obtained sufficient conditions for a normalized analytic functions f to satisfy

$$q_1(z) \prec \frac{f(z)}{zf^{'}(z)} \prec q_2(z)$$

and

$$q_1(z) \prec \frac{z^2f^{'}(z)}{(f(z))^2} \prec q_2(z)$$

where q_1 and q_2 are given univalent functions in $\mathcal{U}$ with $q_1(0) = 1$ and $q_2(0) = 1$.

For two analytic functions $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$, the Hadamard product or convolution of $f(z)$ and $g(z)$, written as $(f * g)(z)$ is defined by

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n \, z^n.$$

Let the function $\phi(b,c;z)$ be given by

$$\phi(b,c;z) = \sum_{n=0}^{\infty} \frac{(b)_n}{(c)_n} z^{n+1} \qquad (c \neq 0, -1, -2, \ldots : z \in \mathcal{U})$$

where $(x)_n$ is the Pochhammer symbol defined, in terms of the Gamma function Γ, by

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = \begin{cases} 1 & if\, n = 0 \\ x(x+1)(x+2)\ \ldots\ (x+n-1) & if\, n \in N = \{1, 2, , \ldots\}. \end{cases}$$

Corresponding to the function $\phi(a, c; z)$, we now define the following operator $D_\lambda^m(a, c)f\ : \mathcal{U} \longrightarrow \mathcal{U}$ by

$$D_\lambda^0(b, c)f(z) = f(z) * \phi(b, c; z)$$

$$D_\lambda^1(b, c)f(z) = (1-\lambda)(f(z) * \phi(b, c; z)) + \lambda\, z(f(z) * \phi(b, c; z))' \tag{1.2}$$

$$D_\lambda^m(b, c)f(z) = D_\lambda^1(D_\lambda^{m-1}(b, c)f(z)). \tag{1.3}$$

If $f \in \mathcal{A}_1$, then from (1.2) and (1.3) we may easily deduce that

$$D_\lambda^m(b, c)f(z) = z + \sum_{n=2}^{\infty} \left[1 + (n-1)\lambda\right]^m \frac{(b)_{n-1}}{(c)_{n-1}}\, a_n z^n, \tag{1.4}$$

where $m \in N_0 = N \cup \{0\}$ and $\lambda \geq 0$. We remark that, for $a = c$ we get the operator recently introduced by F. Al- Oboudi[1], when $b = c$, $\lambda = 1$ we get the operator introduced by G. Ş. Sălăgean [7] and for the choice of the parameter $m = 0$, the operator $D_\lambda^0(b, c)f(z)$ reduces to $L(a, c)$ an operator introduced by Carlson Shaffer [5].
It can be easily verified from the definition of (1.4) that

$$z(D_\lambda^m(b, c)f(z))' = bD_\lambda^m(b+1, c)f(z) - (b-1)\, D_\lambda^m(b, c)f(z) \tag{1.5}$$

and

$$\lambda z(D_\lambda^m(b, c)f(z))' = D_\lambda^{m+1}(b, c)f(z) - (1-\lambda)D_\lambda^m(b, c)f(z). \tag{1.6}$$

The purpose of this paper is to derive the several subordination results involving the operator $D_\lambda^m(b, c)f(z)$. Furthermore, we obtain the results of Shanmugam et al. [8] and Srivastava and Lashin[9] as special cases of some of the results presented here.

In order to prove our subordination and superordination results, we make use of the following known results.

Definition 1.1 *[6] Denote by Q the set of all functions f that are analytic and injective on $\overline{\mathcal{U}} - E(f)$, where*

$$E(f) = \{\zeta \in \partial\mathcal{U} : \lim_{z \to \zeta} f(z) = \infty\},$$

and are such that $f'(\zeta) \neq 0$ for $\zeta \in \partial\mathcal{U} - E(f)$.

Theorem 1.1 *[6] Let the function q be univalent in the open unit disc $\mathcal{U}$ and θ and ϕ be analytic in a domain D containing $q(\mathcal{U})$ with $\phi(w) \neq 0$ when $w \in q(\mathcal{U})$. set $Q(z) = zq^{'}(z)\phi(q(z))$, $h(z) = \theta(q(z)) + Q(z)$. Suppose that*

1. *Q is starlike univalent in $\mathcal{U}$ and*
2. *$Re\left(\frac{zh^{'}(z)}{Q(z)}\right) > 0$ for $z \in \mathcal{U}$.*

If

$$\theta(p(z)) + zp^{'}(z)\phi(p(z)) \prec \theta(q(z)) + zq^{'}(z)\phi(q(z)),$$

then $p(z) \prec q(z)$ and q is the best dominant.

Theorem 1.2 *[4] Let the function q be univalent in the open unit disc $\mathcal{U}$ and ϑ and ϕ be analytic in a domain D containing $q(\mathcal{U})$ Suppose that*

1. *$Re\left(\frac{\vartheta^{'}(q(z))}{\phi(q(z))}\right) > 0$ for $z \in \mathcal{U}$ and*
2. *$zq^{'}(z)\phi(q(z))$ is starlike univalent in $\mathcal{U}$.*

If $p \in \mathcal{H}[q(0), 1] \cap Q$, with $p(\mathcal{U}) \subseteq D$, and $\vartheta(p(z)) + zp^{'}(z)\phi(p(z))$ is univalent in $\mathcal{U}$ and

$$\vartheta(q(z)) + zq^{'}(z)\phi(q(z)) \prec \vartheta(p(z)) + zp^{'}(z)\phi(p(z)),$$

then $q(z) \prec p(z)$ and q is the best subordinant.

2 Subordination And Superordination For Analytic Functions

We begin with the following

Theorem 2.1 *Let $\left(\dfrac{D_{\lambda}^{m}(b+1,c)f(z)}{z}\right)^{\mu} \in \mathcal{H}$ and let the function $q(z)$ be analytic and univalent in $\mathcal{U}$ such that $q(z) \neq 0$, $(z \in \mathcal{U})$. Suppose that $\dfrac{zq^{'}(z)}{q(z)}$ is starlike univalent in $\mathcal{U}$. Let*

$$Re\left\{1 + \frac{\xi}{\beta}\,q(z) + \frac{2\delta}{\beta}(q(z))^{2} - \frac{zq^{'}(z)}{q(z)} + \frac{zq^{''}(z)}{q^{'}(z)}\right\} > 0 \tag{2.1}$$

$$(\alpha,\, \delta,\, \xi,\, \beta \in \mathbb{C};\, \beta \neq 0)$$

and

$$\Psi_\lambda^m(b,c,\mu,\xi,\beta,\delta,f)(z) := \alpha+\xi\left[\frac{D_\lambda^m(b+1,c)f(z)}{z}\right]^\mu+\delta\left[\frac{D_\lambda^m(b+1,c)f(z)}{z}\right]^{2\mu} + \beta\mu(b+1)\left[\frac{D_\lambda^m(b+2,c)f(z)}{D_\lambda^m(b+1,c)f(z)}-1\right]. \tag{2.2}$$

If q satisfies the following subordination:

$$\Psi_\lambda^m(b,c,\mu,\xi,\beta,\delta,f)(z) \prec \alpha+\xi q(z)+\delta(q(z))^2+\beta\frac{zq'(z)}{q(z)},$$

$$(\alpha,\ \delta,\ \xi,\ \mu,\ \beta\in\mathbb{C};\ \beta,\mu\neq 0),$$

then

$$\left(\frac{D_\lambda^m(b+1,c)f(z)}{z}\right)^\mu \prec q(z) \quad (\mu\in\mathbb{C};\ \mu\neq 0) \tag{2.3}$$

and q is the best dominant.

Proof: Let the function p be defined by

$$p(z) := \left(\frac{D_\lambda^m(b+1,c)f(z)}{z}\right)^\mu \qquad (z\in\mathcal{U};\ z\neq 0;\ f\in\mathcal{A})$$

By a straightforward computation, we have

$$\frac{zp'(z)}{p(z)} = \mu\left[\frac{z(D_\lambda^m(b+1,c)f(z))'}{D_\lambda^m(b+1,c)f(z)}-1\right].$$

By using the identity (1.5), we obtain

$$\frac{zp'(z)}{p(z)} = \mu\left[(b+1)\frac{D_\lambda^m(b+2,c)f(z)}{D_\lambda^m(b+1,c)f(z)}-(b+1)\right].$$

By setting

$$\theta(w) := \alpha+\xi\, w+\delta\, w^2 \qquad \text{and} \qquad \phi(w) := \frac{\beta}{w},$$

it can be easily verified that θ is analytic in $\mathbb{C}$, ϕ is analytic in $\mathbb{C}\setminus\{0\}$ and that $\phi(w)\neq 0$ $(w\in\mathbb{C}\setminus\{0\})$. Also, by letting

$$Q(z) = zq'(z)\phi(q(z)) = \beta\frac{zq'(z)}{q(z)}$$

and

$$h(z) = \theta(q(z)) + Q(z) = \alpha + \xi q(z) + \delta(q(z))^2 + \beta \frac{zq^{'}(z)}{q(z)}.$$

We find that $Q(z)$ is starlike univalent in $\mathcal{U}$ and that

$$Re\left(\frac{zh^{'}(z)}{Q(z)}\right) = Re\left\{1 + \frac{\xi}{\beta}\,q(z) + \frac{2\delta}{\beta}(q(z))^2 - \frac{zq^{'}(z)}{q(z)} + \frac{zq^{''}(z)}{q^{'}(z)}\right\} > 0.$$

The assertion (2.3) of Theorem2.1 now follows by an application of Theorem 1.1.

Using arguments similar to those detailed in the Theorem(2.1) with the equation (1.6), we have the following Theorem.

Theorem 2.2 *Let* $\left(\frac{D_\lambda^{m+1}(b,c)f(z)}{z}\right)^{\mu} \in \mathcal{H}$ *and let the function* $q(z)$ *be analytic and univalent in* $\mathcal{U}$ *such that* $q(z) \neq 0$, $(z \in \mathcal{U})$. *Suppose that* $\frac{zq^{'}(z)}{q(z)}$ *is starlike univalent in* $\mathcal{U}$. *Let*

$$Re\left\{1 + \frac{\xi}{\beta}\,q(z) + \frac{2\delta}{\beta}(q(z))^2 - \frac{zq^{'}(z)}{q(z)} + \frac{zq^{''}(z)}{q^{'}(z)}\right\} > 0 \tag{2.4}$$

$$(\alpha,\ \delta,\ \xi,\ \beta \in \mathbb{C};\ \beta \neq 0)$$

and

$$\begin{aligned}\Omega_\lambda^m(b,c,\mu,\xi,\beta,\delta,f)(z) := {} & \alpha + \xi\left[\frac{D_\lambda^{m+1}(b,c)f(z)}{z}\right]^{\mu} + \delta\left[\frac{D_\lambda^{m+1}(b,c)f(z)}{z}\right]^{2\mu} \\ & + \beta\frac{\mu}{\lambda}\left[\frac{D_\lambda^{m+2}(b,c)f(z)}{D_\lambda^{m+1}(b,c)f(z)} - (2-\lambda)\right]\end{aligned} \tag{2.5}$$

If q *satisfies the following subordination:*

$$\Omega_\lambda^m(b,c,\mu,\xi,\beta,\delta,f)(z) \prec \alpha + \xi q(z) + \delta(q(z))^2 + \beta\frac{zq^{'}(z)}{q(z)}$$

$$(\alpha,\ \delta,\ \xi,\ \mu,\ \beta \in \mathbb{C};\ \beta, \mu \neq 0)$$

then

$$\left(\frac{D_\lambda^{m+1}(b,c)f(z)}{z}\right)^{\mu} \prec q(z) \quad (\mu \in \mathbb{C};\ \mu \neq 0) \tag{2.6}$$

and q *is the best dominant.*

For the choices $q(z) = \dfrac{1+Az}{1+Bz}$, $-1 \leq B < A \leq 1$ and $q(z) = \left(\dfrac{1+z}{1-z}\right)^{\gamma}$, $0 < \gamma \leq 1$,in Theorem 2.1, we get the following results.

Corollary 2.3 *Assume that (2.1) holds. If $f \in \mathcal{A}$ and*

$$\Psi_{\lambda}^{m}(b,c,\mu,\xi,\beta,\delta,f)(z) \prec \alpha + \xi\frac{1+Az}{1+Bz} + \delta\left(\frac{1+Az}{1+Bz}\right)^{2} + \beta\frac{(A-B)z}{(1+Az)(1+Bz)}$$

$(\alpha,\ \delta,\ \xi,\ \mu,\ \beta \in \mathbb{C};\ \beta,\mu \neq 0)$, *where $\Psi_{\lambda}^{m}(b,c,\mu,\xi,\beta,\delta,f)$ is as defined in (2.2), then*

$$\left(\frac{D_{\lambda}^{m}(b+1,c)f(z)}{z}\right)^{\mu} \prec \frac{1+Az}{1+Bz} \quad (\mu \in \mathbb{C};\ \mu \neq 0)$$

and $\dfrac{1+Az}{1+Bz}$ *is the best dominant.*

Corollary 2.4 *Assume that (2.1) holds. If $f \in \mathcal{A}$ and*

$$\Psi_{\lambda}^{m}(b,c,\mu,\xi,\beta,\delta,f)(z) \prec \alpha + \xi\left(\frac{1+z}{1-z}\right)^{\gamma} + \delta\left(\frac{1+z}{1-z}\right)^{2\gamma} + \beta\frac{2\gamma z}{(1-z^{2})}$$

$(\alpha,\ \delta,\ \xi,\ \mu,\ \beta \in \mathbb{C};\ \beta,\mu \neq 0)$, *where $\Psi_{\lambda}^{m}(b,c,\mu,\xi,\beta,\delta,f)$ is as defined in (2.2), then*

$$\left(\frac{D_{\lambda}^{m}(b+1,c)f(z)}{z}\right)^{\mu} \prec \left(\frac{1+z}{1-z}\right)^{\gamma} \quad (\mu \in \mathbb{C};\ \mu \neq 0)$$

and $\left(\dfrac{1+z}{1-z}\right)^{\gamma}$ *is the best dominant.*

For special case when $q(z) = \dfrac{1}{(1-z)^{2b}}$ $(b \in \mathbb{C} \setminus \{0\})$, $m = 0$; $a = c = 1$, $\delta = \xi = 0, \mu = \alpha = 1$ and $\beta = \frac{1}{b}$, Theorem 2.1 reduces at once to the following known result obtained by Srivastava and Lashin [9].

Corollary 2.5 *Let b be a non zero complex number. If $f \in \mathcal{A}$, and*

$$1 + \frac{1}{b}\frac{zf^{''}(z)}{f^{'}(z)} \prec \frac{1+z}{1-z},$$

then

$$f^{'}(z) \prec \frac{1}{(1-z)^{2b}}$$

and $\dfrac{1}{(1-z)^{2b}}$, *is the best dominant.*

Remark 2.1 *We remark that Theorem 2.2 can be restated, for different choices of the function q.*

Next, by appealing to Theorem 1.2 of the preceding section, we prove the following:

Theorem 2.6 *Let q be analytic and univalent in $\mathcal{U}$ such that $q(z) \neq 0$ and $\dfrac{zq^{'}(z)}{q(z)}$ be starlike univalent in $\mathcal{U}$. Further, let us assume that*

$$Re\left[\frac{2\delta}{\beta}(q(z))^2 + \frac{\xi}{\beta}q(z)\right] > 0, \quad (\delta,\, \xi,\, \beta \in \mathbb{C};\, \beta \neq 0). \tag{2.7}$$

If $f \in \mathcal{A}$,

$$0 \neq \left(\frac{D_\lambda^m(b+1,c)f(z)}{z}\right)^\mu \in \mathcal{H}\big[q(0),1\big] \cap Q,$$

and $\Psi_\lambda^m(b,c,\mu,\xi,\beta,\delta,f)$ is univalent in $\mathcal{U}$, then

$$\alpha + \xi q(z) + \delta(q(z))^2 + \beta\frac{zq^{'}(z)}{q(z)} \prec \Psi_\lambda^m(b,c,\mu,\xi,\beta,\delta,f)$$

implies

$$q(z) \prec \left(\frac{D_\lambda^m(b+1,c)f(z)}{z}\right)^\mu \quad (\mu \in \mathbb{C};\, \mu \neq 0) \tag{2.8}$$

and q is the best subordinant where $\Psi_\lambda^m(b,c,\mu,\xi,\beta,\delta,f)$ is as defined in (2.2)

Proof: By setting

$$\vartheta(w) := \alpha + \xi\, w + \delta\, w^2 \qquad \text{and} \qquad \phi(w) := \frac{\beta}{w},$$

it can be easily verified that ϑ is analytic in $\mathbb{C}$, ϕ is analytic in $\mathbb{C} \setminus \{0\}$ and that $\phi(w) \neq 0$ $(w \in \mathbb{C} \setminus \{0\})$. Since q is convex (univalent) function it follows that,

$$Re\frac{\vartheta^{'}(q(z))}{\phi(q(z))} = Re\left[\frac{2\delta}{\beta}(q(z))^2 + \frac{\xi}{\beta}q(z)\right] > 0. \quad (\delta,\, \xi,\, \beta \in \mathbb{C};\, \beta \neq 0)$$

The assertion (2.8) of Theorem2.6 follows by an application of Theorem 1.2.

Combining Theorem 2.1 and Theorem 2.6, we get the following sandwich theorem.

Theorem 2.7 *Let q_1 and q_2 be univalent in $\mathcal{U}$ such that $q_1(z) \neq 0$ and $q_2(z) \neq 0$, $(z \in \mathcal{U})$ with $\dfrac{zq_1^{'}(z)}{q_1(z)}$ and $\dfrac{zq_2^{'}(z)}{q_2(z)}$ being starlike univalent. Suppose that q_1 satisfies (2.7) and q_2 satisfies (2.1). If $f \in \mathcal{A}$,*

$$\left(\frac{D_\lambda^m(b+1,c)f(z)}{z}\right)^\mu \in \mathcal{H}\big[q(0),1\big] \cap Q, \quad \text{and} \quad \Psi_\lambda^m(b,c,\mu,\xi,\beta,\delta,f)$$

is univalent in $\mathcal{U}$, then

$$\alpha + \xi q_1(z) + \delta(q_1(z))^2 + \beta\frac{zq_1^{'}(z)}{q_1(z)} \prec \Psi_\lambda^m(b,c,\mu,\xi,\beta,\delta,f)(z)$$

$$\prec \alpha + \xi q_2(z) + \delta(q_2(z))^2 + \beta\frac{zq_2^{'}(z)}{q_2(z)}$$

$(\alpha,\ \delta,\ \xi,\ \mu,\ \beta \in \mathbb{C};\ \beta,\mu \neq 0)$ *implies*

$$q_1(z) \prec \left(\frac{D_\lambda^m(b+1,c)f(z)}{z}\right)^\mu \prec q_2(z) \quad (\mu \in \mathbb{C};\ \mu \neq 0)$$

and q_1 and q_2 are respectively the best subordinant and the dominant.

By taking $m = 0$ and $b = c$ in Theorem 2.7, we have

Corollary 2.8 *Let q_1 and q_2 be univalent in $\mathcal{U}$ such that $q_1(z) \neq 0$ and $q_2(z) \neq 0$, $(z \in \mathcal{U})$ with $\dfrac{zq_1^{'}(z)}{q_1(z)}$ and $\dfrac{zq_2^{'}(z)}{q_2(z)}$ being starlike univalent. Suppose that q_1 satisfies (2.7) and q_2 satisfies (2.1). If $f \in \mathcal{A}$, $(f^{'}(z))^\mu \in \mathcal{H}\big[q(0),1\big] \cap Q$, and let$\Psi(\mu,\xi,\beta,\delta,f) ;= \alpha + \xi\big[f^{'}(z)\big]^\mu + \delta\big[f^{'}(z)\big]^{2\mu} + \frac{3}{2}\beta\mu\frac{zf^{''}(z)}{f^{'}(z)}$ is univalent in $\mathcal{U}$, then*

$$\alpha + \xi q_1(z) + \delta(q_1(z))^2 + \beta\frac{zq_1^{'}(z)}{q_1(z)} \prec \Psi(\mu,\xi,\beta,\delta,f)(z)$$

$$\prec \alpha + \xi q_2(z) + \delta(q_2(z))^2 + \beta\frac{zq_2^{'}(z)}{q_2(z)}$$

$(\alpha,\ \delta,\ \xi,\ \mu,\ \beta \in \mathbb{C};\ \beta,\mu \neq 0)$ *implies*

$$q_1(z) \prec \big(f^{'}(z)\big)^\mu \prec q_2(z) \quad (\mu \in \mathbb{C};\ \mu \neq 0)$$

and q_1 and q_2 are respectively the best subordinant and the dominant.

Remark 2.2 *We note that all the results in [8] follows as a special case of our results.*

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