An. Şt. Univ. Ovidius Constanţa Vol. 15(2), 2007, 71–82

Deterministic multivariate model for simulation of downstream BIVAL automatic controller in irrigation systems

L. Roşu, A. Bărbulescu, S. Hâncu, A. Dumitru

Abstract

The irrigation canals equipped with automatic controllers of the BIVAL type (based on downstream control) are electronic feedback systems. In this paper we present the mathematical model for the unsteady flow in irrigation canals. We give an analytical solution for the unsteady flow equations and for the motion equations of the BIVAL controller.

1 Introduction

Providing irrigation canals with automatic controllers leads to the performance increasing for the following reasons: timely supply of required flow rate at any point of the network, accurate measurements of distributed flow rate, water saving, energy saving, the decreasing of personnel needed in the system exploitation and an increasing efficiency of irrigation system.

A major problem encountered in automatic controlled irrigation canals is to ensure their performance according to the chosen solution.

The importance of solving this problem for the engineering practice is explained by the abundant research done to improve the exploitation performance of existing automatic systems and to diversify the hydraulic calculation methods employed in solving of two important problems: the measurement of water transport capacity and the insurance of hydraulic stability of the network canals during the exploitation.

Key Words: unsteady flow, solution, motion equations
Mathematics Subject Classification: 65N99
Received: April, 2007
Accepted: June, 2007

The majority of the mathematical simulation models are based on the numerical or analytical solution of the nonlinear equations with partial derivatives of hyperbolic type (Saint - Venant equations) that govern the unsteady flow in the canals equipped with the automatic regulators.

In addition to the dynamics and continuity equations of Saint-Venant, there are also some conditions, that describe the hydraulics of the different construction equipments used for canals, the hydraulics of the derivations and of the water distributions points to the clients. The equations of the automatic controllers specific to the chosen solution are also given.

2 The description of the downstream BIVAL automatic controller

The mathematical model describes the behavior of the unsteady flow of water in irrigations canals equipped with electronic controllers of BIVAL type which directly adjust the water level in canal.

The controller's work is based on upstream commands provided by two water level transducers, one placed downstream and the other upstream the pool (Figure 1).

The downstream BIVAL controller helps to deliver the water "based on request" and, as a consequence, the downstream consumer requested rate of flows charts are known.

The design or the exploitation assessment of a pool equipped with such a controller consists in finding the following elements:

- the downstream water level variation chart, $Z_{av}(t)$;
- the rate of flow variation chart at the gate section located upstream the pool, $Q_{am}(t)$;
- the upstream water level variation chart, $Z_{am}(t)$;
- the variation of the upstream gate opening, $a(t)$.

The water elevation Z_i, is given by

$$Z_i = kZ_{am} + (1-k)\, Z_{av}, \tag{2.1}$$

where:

Z_{am} is the water elevation in the upstream transducer section,

Z_{av} - the water elevation in the downstream transducer section,

$k \in [0,1]$ - the weight coefficient.

Based on the value of the weight coefficient k, there are few exploitation situations:

- if $k = 1$, then $L_1 = L$ and $Z_i = Z_{am}$; the controlling section becomes the section where the upstream transducer is located (the adjustment is based on the input coming from downstream the gate),

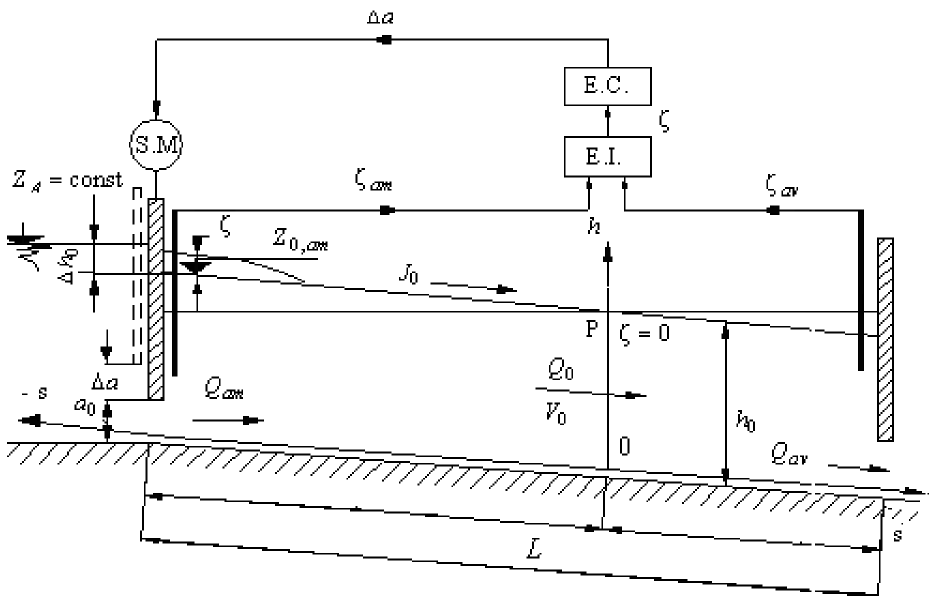

Figure 1: The installation scheme

- if $k = 0$, then $L_2 = L$ and $Z_i = Z_{av}$; the controlling section becomes the section where the downstream transducer is located (the adjustment is based on the input coming from upstream the gate),

- if $k \in (0, 1)$, the controlling section will be located inside the pool between the two transducers.

The hydraulic parameters corresponding to the beginning of the process are:

Z_A = constant - the water elevation in the supply reservoir located upstream the pool,

$Z_{0,\,am}$ - the upstream water elevation (measured by the upstream transducer)

$Z_{0,av}$ - the downstream water elevation (measured by the downstream transducer),

Q_0 - the rate of flow,

h_0 - the water depth in canal,

a_0 - the initial gate opening,

Δh_0 - the hydraulic head of the gate:

$$\Delta h_0 = Z_A - Z_{0,\,am} \tag{2.2}$$

The flow rate under the gate at the moment t_i is given by the equation:

$$Q_{am}^{(t_i)} = c_d \cdot B_s \cdot (a_0 + \Delta a^{(t_i)}) \cdot \sqrt{2g\left(Z_A - Z_{0,am} - \zeta_{am}^{(t_i)}\right)}, \tag{2.3}$$

where:

c_d is the flow rate coefficient of the gate,

B_s is the gate's width.

The rate of flow under the gate at the initial time is:

$$Q_0 = c_d \cdot B_s \cdot a_0 \cdot \sqrt{2g \cdot \Delta h_0} = c_d \cdot B_s \cdot a_0 \cdot \sqrt{2g\,(Z_A - Z_{0,am})}. \tag{2.4}$$

The correction value $\Delta a^{(t_i)}$ which must be made to the gate opening at the moment t_i, as a consequence of the consumption variation, can be found from (2.3), taking into account (2.4):

$$\Delta a^{(t_i)} = \frac{1}{2} \cdot \frac{a_0}{\Delta h_0} \cdot \zeta_{am}^{(t_i)} + \frac{q_{am}^{(t_i)}}{Q_0} \cdot a_0, \tag{2.5}$$

where:

$\zeta_{am}^{(t_i)}$ is the water level oscillations in the upstream section of the pool,

$q_{am}^{(t_i)}$ is the the level oscillations in the upstream section of the pool.

3 The description of the mathematical model

The unsteady flow in the irrigation canals is generated by the rate of flow variations at the water plugs and by the gate maneuvering and is characterized by transverse wave propagation.

To every kind of wave, conceived as physical phenomenon a flow rate waves and a level waves correspond. They are functions of space (s) and time (t), which have perturbed values.

3.1 The fundamental equations of unsteady water flow

The fundamental equations of unsteady water flow in open canals are the Saint-Venant equations: the dynamic equation and the continuity equation. They form a system of nonlinear equations with variable coefficients, which belongs to the hyperbolic differential equations family:

$$\begin{cases} \dfrac{\partial Z}{\partial s} + \dfrac{1}{g} \cdot \dfrac{\partial V}{\partial t} + \dfrac{1}{g} \cdot \dfrac{V \cdot \partial V}{\partial s} + \dfrac{Q^2}{K^2} = 0 \\ \dfrac{\partial A}{\partial t} + \dfrac{\partial Q}{\partial s} = 0 \end{cases} \tag{3.1}$$

In these two equations the following hydraulic parameters are functions of space (s) and time (t):

- $Z = Z(s,t)$ - the free water elevation,
- $Q = Q(s,t)$ - the flow rate in the flow section,
- $V = V(s,t)$ - the flow velocity,
- $J = J(s,t)$ - the hydraulic slope of the stream ($J = Q^2/K^2$),
- $A = A(s,t)$ - the flow cross section area,
- $K = A \cdot C \cdot \sqrt{R}$ - the flow rate coefficient,
- $C = R^y/n$ - the Chezy resistance coefficient,
- n - the roughness coefficient,
- y - exponent (in Manning's relation, $y = 1/6$)
- g - the gravity acceleration.

Remark. The general solutions of the equations (3.1) will be denoted by $q(s,t)$ and $\zeta(s,t)$.

3.2 The linearized equations of the unsteady flow

Among the different forms of the wave functions, a special interest presents the function which varies exponentially with the phase and corresponds to the harmonic plane waves.

Although the harmonic plane wave is an idealized concept, since in nature there are no such waves, it is a good model approach for our study. First of all, because any perturbation - no matter how complicated - can be described, using the Fourier integral, as a sum of elementary perturbations and the propagation of each elementary perturbation can be described by an harmonic wave. Secondly, in the conditions of the linear form of the motion equations, the wave functions of the harmonic waves, being written in exponential form, are easy to use in computations and the reconstruction of the original wave can be obtained by superposition the elementary waves (harmonics).

Using this concept and taking into account that in practical engineering problems (for design or exploitation of automatic controlled open canals) the variation charts of the flows rate consumption and of the water level are described by periodical functions, the next step is to make an harmonic analyze of the flow rate function, $Q(t)$ and of the level function, $Z(t)$, in order to substitute them by finite sums of simple harmonics.

Remark 1. If we fix a calculus section, then $Q(s,t)$ will be denoted by $Q(t)$.

On the other hand, using the main hypothesis of small oscillation theory (all hydraulic parameters of the oscillatory motion are small and consequently their squared values and their products are small in comparison to all the parameters containing as factor a perturbation parameter), the non linear Saint-Venant equations with variable coefficients are transformed in linear equations of second order with constant coefficients (the initial state of motion was assumed to be permanent).

Remark 2. The linear equations of the motion and their solutions $q(t)$ and $\zeta(t)$ are presented bellow for two situations:

-the flow direction is the same with the s axis (Figure 2):

$$\frac{\partial^2\zeta}{\partial t^2} - \left(c^2 - V_0^2\right)\frac{\partial^2\zeta}{\partial s^2} + 2V_0 \cdot \frac{\partial^2\zeta}{\partial s\partial t} + \alpha\frac{\partial\zeta}{\partial t} + \beta\frac{\partial\zeta}{\partial s} = 0, \tag{3.2}$$

$$B_0 \cdot \frac{\partial\zeta}{\partial t} + \frac{\partial q}{\partial s} = 0. \tag{3.3}$$

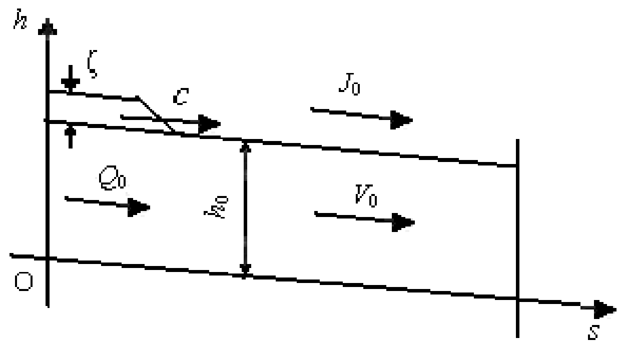

Figure 2: The positive flow direction

-the flow direction is opposite to the s axis (Figure 3):

$$\frac{\partial^2\zeta}{\partial t^2} - \left(c^2 - V_0^2\right)\frac{\partial^2\zeta}{\partial s^2} - 2V_0 \cdot \frac{\partial^2\zeta}{\partial s\partial t} + \alpha\frac{\partial\zeta}{\partial t} + \beta\frac{\partial\zeta}{\partial s} = 0, \tag{3.4}$$

$$B_0 \cdot \frac{\partial\zeta}{\partial t} + \frac{\partial q}{\partial s} = 0. \tag{3.5}$$

In the above equations, c is the celerity, B_0 is the canal width at the free water level, α and β are coefficients depending on the geometry of the canal and its hydraulic parameters. The subscript zero is given to the reference (initial) values of the parameters.

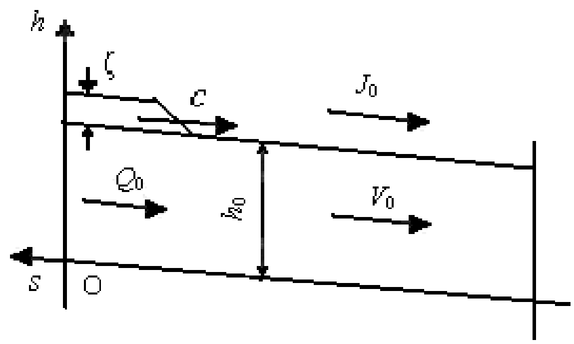

Figure 3: The negative flow direction

3.3 The general solutions of the unsteady flow equations

To find the general solutions of the motion equations, we determine ζ from the equations (3.2), (3.4) and q from the equations (3.3) and (3.5), namely:

- When the flow direction is the same as the s axis direction, the general solution of the dynamics equation is:

$$\zeta(s,\, t) = a_1 \cdot e^{p_1 s} \cdot \cos(\omega t + q_1 s + \varphi_1) + a_2 \cdot e^{p_2 s} \cdot \cos(\omega t + q_2 s + \varphi_2). \quad (3.6)$$

Similarly, the general solution of the continuity equation is:

$$q\,(s,t) = B_0 \cdot \omega \cdot \frac{a_1 \cdot e^{p_1 s}}{\rho_1} \sin(\omega t + q_1 s + \varphi_1 - \psi_1) +$$

$$+ B_0 \cdot \omega \cdot \frac{a_2 \cdot e^{p_1 s}}{\rho_2} \sin(\omega t + q_2 s + \varphi_2 - \psi_2). \quad (3.7)$$

- When the flow direction is opposite to the s axis, the general solution of the dynamics equation is:

$$\zeta(s,\, t) = a_1 \cdot e^{-p_2 s} \cdot \cos(\omega t - q_2 s + \varphi_1) + a_2 \cdot e^{-p_1 s} \cdot \cos(\omega t - q_1 s + \varphi_2). \quad (3.8)$$

Similarly, the general solution representing the rate of flow, $Q(s,t)$, is:

$$q\,(s,t) = -B_0 \cdot \omega \cdot \frac{a_1 \cdot e^{-p_2 s}}{\rho_2} \sin(\omega t - q_2 s + \varphi_1 - \psi_2) +$$

$$- B_0 \cdot \omega \cdot \frac{a_2 \cdot e^{p_1 s}}{\rho_1} \sin(\omega t - q_1 s + \varphi_2 - \psi_1). \quad (3.9)$$

In the equations (3.6) - (3.9), a_1, a_2, φ_1, φ_2 are integration constants which can be found using the initial boundary conditions and the boundary conditions related to the BIVAL controller. ω is the oscillation frequency, concordantly with the flow rate period and the level functions. The terms p_1, p_2, q_1, q_2, ψ_1, ψ_2, ρ_1, ρ_2 are parameters depending on the geometry and hydraulics of the canal.

3.4 The solutions determination for the motion equations of the BIVAL controller

For the BIVAL type controller, the general solutions for the unsteady flow are easier to use if a convenient reference system is chosen such that the ordinate axe passes through the point P dividing the pool of length L in two calculation sections as follows:

- section I of length L_1, between P and the downstream end of the pool, for which the s axis has the same direction as the flow ($L_1 = kL$),

- section II of length L_2, between P and the upstream end of the pool, for which the s axis has opposite direction to the flow ($L_1 = (1-k)L$.)

To simplify the calculations it was assumed that at the point P the water elevation remains constant for small rate of flow variations. The assumption is reasonable for the linearized equations of the motion. The matching condition of the two calculation sections is expressed using $q_0(t)$ and $\zeta_0(t)$ functions at the point P where $s = 0$.

Keeping the same notations used till now to avoid any further confusion, the pair of functions representing the harmonic oscillations for the level $\zeta(t)$ and the rate of flow $q(t)$ in a given section are denoted as follows:

- for the section $s = 0$ (at the point P): $\zeta_0(t)$ and $q_0(t)$,

- for the section downstream the pool: $\zeta_L(t) = \zeta_{av}(t)$ and $q_L(t) = q_{av}(t)$,

- for the section upstream the pool: $\zeta_{kL}(t) = \zeta_{am}(t)$ and $q_{kL}(t) = q_{am}(t)$.

The general solutions given in ((3.6) and (3.7)) or ((3.8) and (3.9)) are used to find the integration constants, considering the boundary conditions for the two calculation sections:

a) The calculation section I has the following boundary conditions:

- $\zeta_0(t) = 0$, that expresses the matching condition,

- the function $q_L(t)$, that expresses the downstream consumption variation

.

The unknown functions are:

-$L(t)$, that describes the harmonic oscillations of the water level downstream the pool (at the consumer),

- $q_0(t)$, that describes the rate of the flow oscillations at the point P.

b) The calculation section II has the following boundary conditions:

- $\zeta_0(t) = 0$,
- the function $q_0(t)$, which was found in the previous calculation phase.

The unknown functions are:

- $\zeta_{Lk}(t)$, describing the harmonic oscillations at the upstream controlled gate section (in the adopted sens for the s axis orientation, the upstream section of the pool corresponds to the downstream section of the pool for section II),
- $\zeta_{Lk}(t)$, describing the variations of the upstream delivered rate of the flow which becomes the flow rate through the gate

Referring to the whole pool, the boundary conditions for section II are the matching conditions at the point P. The condition imposed at P, $\zeta_0(t) = 0$, makes Euler-Fourier coefficients to become zero for the harmonics of the zero order and also for those terms containing them.

The integration constants a_1, a_2, φ_1, φ_2 found in this way have the following expressions:

$$a_1 = \frac{\rho_1 \cdot \rho_2 \cdot W_0}{B_0 \cdot \omega \cdot D_{oP}}\ ,\ a_2 = a_1, \tag{3.10}$$

$$\begin{matrix} \varphi_1 = \arcsin\left(\dfrac{\rho_2 \cdot P_0 - \rho_1 \cdot R_0}{W_0}\right) \\ \varphi_2 = \arccos\left(\dfrac{\rho_2 \cdot M_0 - \rho_2 \cdot N_0}{W_0}\right) \end{matrix}\ ,\ \left\{ \begin{matrix} \varphi_1 = \arcsin\left(\dfrac{\rho_1 \cdot R_0 - \rho_2 \cdot P_0}{W_0}\right) \\ \varphi_2 = \arccos\left(\dfrac{\rho_2 \cdot N_0 - \rho_2 \cdot M_0}{W_0}\right) \end{matrix} \right. \tag{3.11}$$

In the equations (3.10) and (3.11) the terms M_0, N_0, P_0, R_0, W_0, D_0 contain the harmonics coefficients which are referring to the delivered rate of the flow upstream the pool and the damping factors for the oscillation propagation along the pool.

After the determination of integration constants a_1, a_2, φ_1, φ_2, by harmonic synthesis for the harmonics of j order, the unknown functions (representing the level oscillations upstream the pool $\zeta_{am}(t)$ and the rate of flow oscillations $q_{am}(t)$) can also be found. The correction value which must be made to the gate opening, $\Delta a(t)$, representing the gate command element, is obtained using the relation (2.5).

If $q_{av}^{(t_i)}$ is a flow rate perturbation downstream the pool occurring at a moment t_i, due to the variation of flow rate consumption, then the hydraulic parameters at the moment t_i are:

-for the pool upstream section the gate section):

$$Q_{av}^{(t_i)} = Q_0 + q_{av}^{(t_i)} \quad \text{(the delivered rate of flow)}, \tag{3.12}$$

$$Z_{am} = Z_{0,\,am} + \zeta_{am}^{(t_i)} \quad \text{(the water elevation)}$$

or

$$h_{am}^{(t_i)} = h_0 + \zeta_{am}^{(t_i)} \quad \text{(the water height)}, \tag{3.13}$$

$$a^{(t_i)} = a_0 + \Delta a^{(t_i)} \text{ (the gate opening)}, \tag{3.14}$$

where:

- $q_{am}^{(t_i)}$ is the perturbation of the flow delivered rate in the pool upstream section,
- $\zeta_{am}^{(t_i)}$ - the level oscillations in the pool upstream section,
- $\Delta a^{(t_i)}$ - the correction that must be made to the gate opening.
- for the pool downstream section (the consumer section):

$$Q_{av}^{(t_i)} = Q_0 + q_{av}^{(t_i)} \quad \text{(the consumed rate of flow)}, \tag{3.15}$$

$$Z_{av} = Z_{0,\,av} + \zeta_{av}^{(t_i)} \quad \text{(the water elevation)}$$

$$h_{av}^{(t_i)} = h_0 + \zeta_{av}^{(t_i)} \quad \text{(the water height)}, \tag{3.16}$$

where:

- $q_{av}^{(t_i)}$ is the perturbation of the flow consumed rate in the pool downstream section,
- $\zeta_{av}^{(t_i)}$ - the level oscillations in the pool downstream section.

The relations (3.12) - (3.16) allow to the design and assessment of the irrigation canals equipped with BIVAL type controllers. They also allow to find the variation charts for the delivered and consumed flow rate, the variation chart of water level in a canal and the variation chart of gate opening.

4 Conclusions

Sometimes, in the research literature is mentioned that solving the non linear Saint-Venant equation system for unsteady flow by direct integration can not be done unless one considers particular cases which present no practical interest.

The present paper contradicts such a statement, offering an answer on how to conceive and use an analytical model for the design of the automatic irrigation canals, not for a particular case, but for practical important situations.

Knowing the large use of automatic controllers (with upstream, downstream or BIVAL command) on large capacity canals, the problem becomes important both for theoretical research and practical exploitation situations.

In this paper was described a model for the automatic system of the BIVAL controller and canal.

The model was obtained by analytical integration of linearized equations based on the hypothesis of small oscillation theory and the properties of the

Fourier transforms. This is a dynamical model, its hydraulic parameters (the water level Z, the rate of flow Q, the gate opening a) are functions of time, the variables input generate output variables and thus, it describes the system's behavior in time.

Since the described physical phenomenon is governed by well defined equations and the perturbations induced in system can be determined, the model is deterministic and multivariate.

Its purpose is to predict the behavior of the system under the perturbation factors and to help in the decision taking process.

The modeling basic principles have been accomplished by: the accurate simulation of the physical phenomenon, flexibility, adaptability to exploitation situations, a clear concept. The physical parameters can be easily determined and the system can be solved by automatic computation.

References

[1] S. Hâncu, E. Rus, P. Dan, Gh. Teodoreanu, *The hydraulics of the automatic irrigation systems*(in Romanian), CERES, Bucharest, 1982.

[2] M. D. Certousov , *Hydraulics*(in Romanian), Ed. Tehnică, Bucharest, 1966.

[3] L. Rosu, *Design and assessment of the automatic irrigation systems*(in Romanian), "Ovidius" University Press, Constantza, 1999.

[4] *** Research Contract No. 3400/ 1999, Grant Convention CNCSIS no. 169, "Ovidius" University of Constantza (in Romanian)

L. Rosu
"Ovidius" University of Constanta
The Faculty of Civil Engineering,
900527 Constanta, Bd. Mamaia 124
Romania
e-mail: lrosu@univ-ovidius.ro

A. Barbulescu
"Ovidius" University of Constanta
Department of Mathematics and Informatics,
900527 Constanta, Bd. Mamaia 124
Romania
e-mail: abarbulescu@univ-ovidius.ro

S Hâncu
U.A.S.V.M. Bucharest, Romania
The Faculty of Land Reclamation and Environment Engineering

A. Dumitru
Shriners Burns Hospital Research Department, Boston, US